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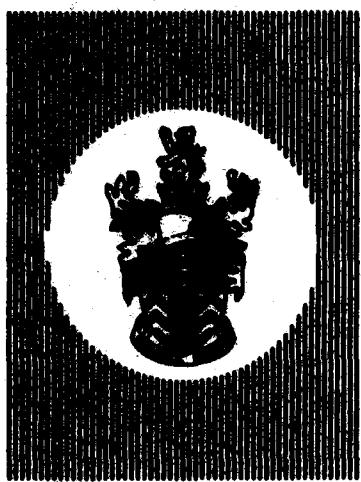
A UNIFIED TREATMENT OF THE THEORY OF SIX—PORT
REFLECTOMETER CALIBRATION USING THE MINIMUM
OF STANDARDS

Authors: T E Hodgetts
and E J Griffin

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SUMMARY

This report presents a unified treatment of theory for
calibrating the six-port reflectometer and the dual reflectometer
network analyser for measuring power and scattering coefficients,
given the minimum of standards for each. It describes a reflecto-
meter in terms of voltage variables and reduces the calibration
theory to equations in real unknowns to facilitate encoding the
computation necessary for known calibration methods in a low-level
computer language. The treatment forms the basis for software in
use with six-port reflectometers and dual six-port network analysers
in development at RSRE for comparing RF and microwave metrology
standards.

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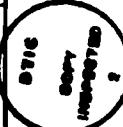
RSRE REPORT NO 83003

A UNIFIED TREATMENT OF THE THEORY OF SIX-PORT REFLECTOMETER CALIBRATION USING THE MINIMUM OF STANDARDS

T E Hodgetts and E J Griffin

1 INTRODUCTION

Measurement of power and complex voltage transmission and reflection coefficients with a six-port reflectometer were first described by Hoer and Engen more than a decade ago (1,2). This form of reflectometer consists essentially of four radiation detectors, each indicating the power absorbed by it, arranged around a structure guiding radiation to the detectors and to the device under test (DUT). The waveguide structure serves solely to direct to the detectors different samples of waves incident on and reflected from the DUT and observations are made of the ratios of power absorbed by three of the detectors to that absorbed by the fourth. This simplicity of principle implies that this form of reflectometer may be used for measuring scattering coefficients at any frequency at which artefacts can provide physical approximation to a conceptual impedance for calibrating the instrument. Examples have been reported operating over the 10 MHz to 1 GHz range by Woods and Granville-George(3), from 2 to 18 GHz by Hoer and Engen (4), and up to 75 GHz by Weidman (5).

This detailed report presents a unified treatment of the theory of known procedures for calibrating both the six-port reflectometer and the dual reflecto-meter network analyser (6-8). Because on-line computation of results is an operational necessity, the theory has been reduced to equations in real unknowns without ambiguity of square-root signs to facilitate programming in a low-level computer language. The treatment was developed as a necessary prerequisite for programming desktop computers controlling these instruments for the eventual aim of comparing standards for RF and microwave metrology. To meet this aim the calibration methods treated are mainly, but not exclusively, those requiring the minimum of known standards.

Although the operation of six-port instruments depends on the observation of power ratios, we start from a description of an n-port reflectometer in terms of voltage variables. This is appropriate because a scattering coefficient is merely a ratio of two complex numbers each representing a voltage. This description also has the merit of enabling the relatively few assumptions under-lying six-port reflectometer use to be identified. Such identification is as necessary for quantifying uncertainty of measurement with a six-port as it is for any other form of reflectometer.

Starting from a description of the n-port reflectometer also facilitates establishing equivalence relations between six- and four-port reflectometers. These relations enable two six-port reflectometers forming a network analyser to be calibrated in terms of a reflectionless waveguide plus a reflecting termina-tion of unknown but repeatable voltage reflection coefficient (VRC). This method (8) calibrates both reflectometers in terms of the impedance Z_0 character-ising the mode transmitted by a known length of uniform waveguide (reflectionless by definition) and reflected by the termination. It yields the propagation constant of the waveguide and if this be air-dielectric transmission line then subsequently measured results can be normalised to the nominal real R_0 ($\equiv \mathcal{R} Z_0$) characterising the TEM mode postulated for a lossless transmission line.

2 INTRODUCTION TO SIX-PORT REFLECTOMETER CALIBRATION

2.1 THE GENERAL N-PORT (n > 3) REFLECTOMETER

We define a reflectometer as an n-port linear, but not necessarily reciprocal, time-stationary waveguide junction (where n > 3) directing radiation from a source at port 1, Figure 1, to an output (measurement)
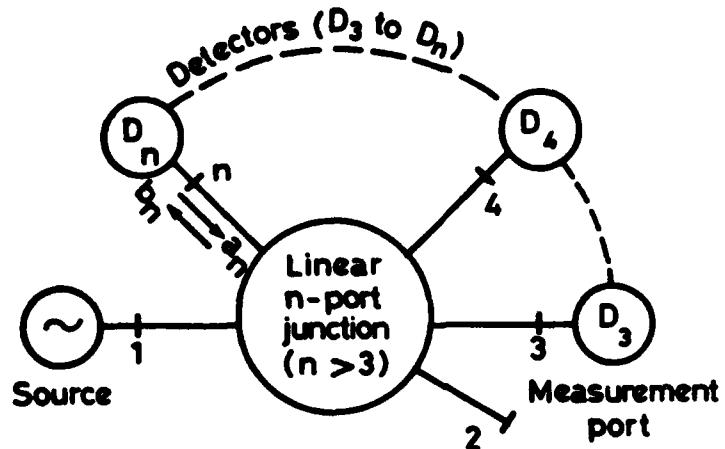
FIG. 1. GENERAL n-PORT REFLECTOMETER (n>3)

port 2 and to detectors at ports 3 to n. We assume that there is one mode transmitted at each port but not necessarily the same mode at all ports and that there is no evanescent mode at any port; these assumptions enable us to include mode transformers within the junction. They also enable us to define at the reference plane of each port i (where $n \geqslant i > 0$) voltages proportional to the electric field intensity described by the transmitted mode of waves incident on and emergent from port i. We denote these by complex numbers a_i and b_i, respectively, such that their RMS amplitudes are $|a_i|$ and $|b_i|$. The assumption of junction linearity implies that each b_i is composed of simply the sum of contributions each proportional to an a_i and this can be conveniently represented by the matrix notation

$$[b] = [S][a] \qquad\qquad (2.1)$$

in which the elements of each row of [S] are complex numbers relating the corresponding b_i of the column matrix [b] to every a_i forming the column matrix [a]. The elements of [S] can be considered as scattering coefficients normalised to a yet-to-be-defined impedance.

Without defining this impedance we can define for each detector connected to port i (where now $n \geqslant i > 2$) a VRC $\Gamma_i = a_i/b_i$. These Γ_i can be substituted in equation (2.1) to give

$$
\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ \vdots \\ b_5 \end{bmatrix} = [S] \begin{bmatrix} a_1 \\ a_2 \\ \Gamma_3 b_3 \\ \Gamma_4 b_4 \\ \vdots \\ \Gamma_n b_n \end{bmatrix}
$$

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For $n \geqslant i > 2$ this notation represents

$$b_i = s_{i1}a_1 + s_{i2}a_2 + s_{i3}\Gamma_3 b_3 + \ldots + s_{ii}\Gamma_i b_i + \ldots + s_{in}\Gamma_n b_n$$

$$\therefore \quad 0 = s_{i1}a_1 + s_{i2}a_2 + s_{i3}\Gamma_3 b_3 + \ldots + (s_{ii}\Gamma_i - 1)b_i + \ldots + s_{in}\Gamma_n b_n$$

so that

$$
\begin{bmatrix} b_1 \\ b_2 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}
=
\begin{bmatrix}
s_{11} & s_{12} & s_{13}\Gamma_3 & s_{14}\Gamma_4 & s_{1n}\Gamma_n \\
s_{21} & s_{22} & s_{23}\Gamma_3 & s_{24}\Gamma_4 & s_{2n}\Gamma_n \\
s_{31} & s_{32} & (s_{33}\Gamma_3 - 1) & s_{34}\Gamma_4 & s_{3n}\Gamma_n \\
s_{41} & s_{42} & s_{43}\Gamma_3 & (s_{44}\Gamma_4 - 1) & s_{4n}\Gamma_n \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
s_{n1} & s_{n2} & s_{n3}\Gamma_3 & s_{n4}\Gamma_4 & (s_{nn}\Gamma_n - 1)
\end{bmatrix}
\begin{bmatrix} a_1 \\ a_2 \\ b_3 \\ b_4 \\ \vdots \\ b_n \end{bmatrix}
\qquad (2.2)
$$

We now make the further assumption that there is unequal transmission between the ports of the junction (a condition hard to avoid in practice). This implies that, denoting the square matrix of equation (2.2) by $[\overline{M}]$, the elements of any row of $[\overline{M}]$ are not in general multiples of the corresponding elements of any other row so that $\det[\overline{M}] \neq 0$. Thus $[\overline{M}]$ may in general be inverted to give $[M] = [\overline{M}]^{-1}$ and we can write

$$
\begin{bmatrix} a_1 \\ a_2 \\ b_3 \\ b_4 \\ \vdots \\ b_n \end{bmatrix}
= [M]
\begin{bmatrix} b_1 \\ b_2 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}
$$

or its equivalent

$$a_1 = m_{11}b_1 + m_{12}b_2 \qquad (2.3.1)$$

$$a_2 = m_{21}b_1 + m_{22}b_2 \qquad (2.3.2)$$

$$b_i = m_{i1}b_1 + m_{i2}b_2 \quad \text{(for } n \geqslant i > 2\text{)} \qquad (2.3.3)$$

where m_{i1} and m_{i2} denote elements of the first and second columns of $[M]$, respectively.

Rewriting equation (2.3.2) as $b_1 = (a_2 - m_{22}b_2)/m_{21}$ and substituting in equation (2.3.3) gives

$$b_i = \frac{m_{i1}}{m_{21}}a_2 + \left(m_{i2} - \frac{m_{22}m_{i1}}{m_{21}}\right)b_2$$

Defining $\Gamma \equiv a_2/b_2$ as the VRC of an artefact connected to port 2 (either a DUT or a calibration standard) allows us to write from this equation

$$b_i = \left(\frac{m_{i1}}{m_{21}} \Gamma + \left(m_{i2} - \frac{m_{22}m_{i1}}{m_{21}}\right)\right)b_2 \qquad (2.4)$$

or

$$b_i = (\alpha_i \Gamma + \beta_i)b_2 \qquad (2.5)$$

where $\alpha_i = m_{i1}/m_{21}$ and $\beta_i = (m_{i2}m_{21} - m_{22}m_{i1})/m_{21}$ are dimension-less constants characterising the waveguide junction.

Equations (2.4) and (2.5) are fundamental to all reflectometers comprising a time-invariant waveguide junction. Their application to varying junctions is dependent on the validity of additional assumptions such as the repeatability of RF switches within the junction. Application of either equation to a particular reflectometer depends on the method of detection employed since it is the detector output, or some function of it, that forms the observed indication. In the next section we apply these equations to a four-port reflectometer to illustrate this dependence on detector output and to establish an equivalence between the four-port and a perfect reflectometer preceded by a two-port "error box", a representation we later use.

2.2 THE FOUR-PORT REFLECTOMETER

For this $n = 4$ in Figure 1, and the two detectors D_3 and D_4 are each assumed to provide an output voltage v_i ($i = 3,4$) proportional to the RF voltage presented to it by the four-port waveguide junction. These voltages v_i are at an IF to enable both the real and imaginary components of v_4/v_3 to be indicated by an associated superheterodyne receiver (9). We initially assume that the complex indication provided by the receiver is some function of this ratio, ie indication $w = fn(v_4/v_3)$. We relate w to the b_3 and b_4 of equation (2.5) by first writing

$$v_i = (a_i + b_i) = (1 + \Gamma_i)b_i$$

and substituting in (2.5) to give

$$v_i = (1 + \Gamma_i)(\alpha_i \Gamma + \beta_i)b_2$$

$\therefore \qquad w = fn(v_4/v_3) = fn((1 + \Gamma_4)(\alpha_4 \Gamma + \beta_4)/((1 + \Gamma_3)(\alpha_3 \Gamma + \beta_3))) \qquad (2.6)$

or $\qquad w = fn((d\Gamma + e)/(c\Gamma + 1))$

where $\qquad c = \alpha_3/\beta_3$, $d = (1 + \Gamma_4)\alpha_4/((1 + \Gamma_3)\beta_3)$

$\qquad e = (1 + \Gamma_4)\beta_4/((1 + \Gamma_3)\beta_3)$

are characteristic of the reflectometer.

If the additional assumption be made that the receiver indication w is a linear function of v_4/v_3, absorbing the coefficients of linearity in c,d,e equation (2.6) becomes the bilinear transformation between w and Γ given by

$$w = (d\Gamma+e)/(c\Gamma+1) \qquad (2.7)$$

Physical significance is imparted to w, enabling Γ to be found from equation (2.7), only by calibrating the instrument. Because it is characterised by three complex numbers c,d,e, it can be calibrated by observing w_ℓ ($\ell = 1,2,3$) corresponding to the connection of three standards providing different known Γ_ℓ. Therefore it is the impedance in terms of which these Γ_ℓ are known that defines the Z_0 to which

    i.   the result of measurement Γ,

    ii.   the elements of [S] in equation (2.1), and

    iii.   the Γ_i of the detectors

are all normalised. It is this Z_0 that determines c,d,e in the bilinear transformation (2.7) to completely describe the reflectometer (comprising the four-port junction, detectors and, given linearity of response, the associated receiver).

The explanation in the preceding paragraph, that the impedance left undefined in section 2.1 is defined by the Z_0 to which the Γ_ℓ of known calibration standards is normalised, applies equally to any reflectometer. It will be shown in section 3 that a modification of the calibration method just outlined can be applied to the six-port reflectometer, so we proceed to derive theory appropriate to four-port reflectometer calibration for later use. Writing for at least three known VRC $\Gamma_\ell = x_\ell + jy_\ell$ ($j \equiv \sqrt{-1}$; $\ell = 1,2,3$) and, for the corresponding $w_\ell = u_\ell + jv_\ell$, and denoting the real and imaginary components of c,d,e by suffixes R and I respectively, then equation (2.7) becomes

$$(u_\ell + jv_\ell)\{1+(c_R+jc_I)(x_\ell+jy_\ell)\} = (d_R+jd_I)(x_\ell+jy_\ell) + (e_R+je_I) \quad (2.8)$$

Separating real from imaginary components gives for $\ell = 3$ an exact, and for $\ell > 3$ an overdetermined, set of simultaneous linear equations in the six real numbers constituting c,d,e:

$$\left.\begin{aligned}
u_\ell &= d_R x_\ell - d_I y_\ell + e_R - (x_\ell u_\ell - y_\ell v_\ell)\,c_R + (x_\ell v_\ell + y_\ell u_\ell)c_I \\
v_\ell &= d_R y_\ell + d_I x_\ell + e_I - (x_\ell v_\ell + y_\ell u_\ell)\,c_R - (x_\ell u_\ell - y_\ell v_\ell)c_I
\end{aligned}\right\} \quad (2.9)$$

After equations (2.9) have been solved for c,d,e (using a least-squares method if more than the minimum of standards are used), they can be recast to allow the VRC of a subsequently connected DUT to be found from

$$\left.\begin{aligned}
x &= \{p(u-e_R) - q(v-e_I)\}/(p^2+q^2) \\
\\
y &= \{q(u-e_R) + p(v-e_I)\}/(p^2+q^2)
\end{aligned}\right\} \quad (2.10)$$

where $p = (d_R-uc_R+vc_I)$ and $q = (-d_I+vc_R+uc_I)$

Thus any unknown $\Gamma = x+jy$ can in principle be found from the complex
indication w provided that p and q are not both zero (which we show later
to be true generally). The words "in principle" have been used because
measurement relies on (2.7) being used to interpolate in both x and y
directions in the complex Γ plane between the known $\Gamma_\ell = x_\ell + jy_\ell$ provided
by the standards; extrapolation beyond those values may not be valid
(because, for example, the detector response may no longer be linear).

An alternative treatment of the four-port reflectometer regards it as
equivalent to an "error box" (ie a two-port junction having equivalence to
c,d,e), one port of which forms the measurement port and having connected
to the other a perfect reflectometer (one whose indication w is simply a
VRC) as shown in Figure 2.
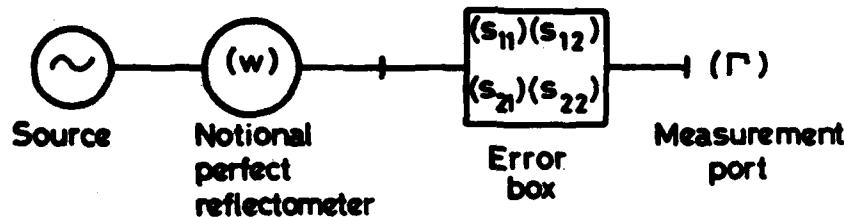
FIG.2. ERROR BOX REPRESENTATION OF REFLECTOMETER

We derive this equivalence for later use by remembering that a two-port
junction with scattering matrix

$$\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}$$

transforms a VRC Γ into w, where

$$w = s_{11} + \frac{s_{12}s_{21}\Gamma}{1 - s_{22}\Gamma}$$

$$= \left(s_{11} - \frac{s_{12}s_{21}}{s_{22}} \right) + \frac{s_{12}s_{21}}{s_{22}(1-s_{22}\Gamma)}$$

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Comparing this with equation (2.7) rewritten as

$$w = \frac{d}{c} + \frac{e-d/c}{c\Gamma+1} \tag{2.11}$$

shows the equivalence of the "error box" representation to be valid if

$$c = -s_{22} \quad , \quad d = s_{12}s_{21} - s_{11}s_{22} \quad , \quad e = s_{11}$$

Both representations account for RF signal leakage between the detectors, since transmission between all ports of the waveguide junction was initially postulated; neither takes account of any interdependence of phase and amplitude within the associated receiver. (We remark that they may be applied to the reflectometers of a dual reflectometer network analyser (see section 4) only if there are no leakage paths between the reflectometers.)

The error box representation makes it evident that the reflectometer cannot function if $s_{12}s_{21} = 0$. We may therefore assert that $s_{12}s_{21} \neq 0$ and this is equivalent to $d-ce \neq 0$. But from equation (2.10)

$$p - jq = d_R + jd_I - (c_R+jc_I)(u+jv)$$

so that $p^2+q^2 = |p-jq|^2 = |d-wc|^2$

Hence the condition $p = q = 0$ is equivalent to $w = d/c$ in (2.11) which is inadmissible since $d-ce \neq 0$. Thus, as was asserted earlier, $\Gamma = x+jy$ can always be found from w.

2.3 THE SIX-PORT REFLECTOMETER

For this, $n = 6$ in Figure 1, and each of the four detectors D_3 to D_6 is assumed to provide a linear indication of the power it absorbs. (In practice bolometric detectors are most frequently used because of their excellent approximation to this assumption (10)). If each indicates power $P_i = G_i(|b_i|^2 - |a_i|^2) = G_i(1-|\Gamma_i|^2)|b_i|^2$, (where $6 > i > 2$) then G_i may be interpreted as the conductive component of an admittance Y_i characterising the mode transmitted at port i. From equation (2.4) we form

$$|b_i|^2 = \left| \frac{m_{i1}}{m_{21}} \Gamma + \left(m_{i2} - \frac{m_{22}m_{i1}}{m_{21}} \right) \right|^2 |b_2|^2$$

so that

$$P_i = G_i(1-|\Gamma_i|^2) \left| \frac{m_{i1}}{m_{21}} \Gamma + \left(m_{i2} - \frac{m_{22}m_{i1}}{m_{21}} \right) \right|^2 |b_2|^2$$

$$\therefore \quad P_i = \quad \alpha_i^2 \quad + \quad \beta_i^2 \quad |b_2|^2 \tag{2.12}$$

where α_i and β_i are defined by $\alpha_i^2 = G_i(1-|\Gamma_i|^2)(m_{i1}/m_{21})^2$

and $\beta_i^2 = G_i(1-|\Gamma_i|^2)((m_{i2}m_{21} - m_{22}m_{i1})/m_{21})^2$

by analogy with the definitions at (2.5).

Defining $|w_k|^2 = P_i/P_3$ for $6 > i > 3$ and $k = i - 3$ gives

$$|w_k|^2 = \left|\frac{\alpha_i\Gamma+\beta_i}{\alpha_3\Gamma+\beta_3}\right|^2$$

or $\qquad |w_k|^2 = \left|\frac{d_k\Gamma+e_k}{c\Gamma+1}\right|^2$ $\hspace{3cm}$ (2.13)

where $\qquad c = \alpha_3/\beta_3$, $d_k = \alpha_i/\beta_3$, $e_k = \beta_i/\beta_3$

From (2.13) the three observed power ratios can be interpreted as defining three circles in the complex plane intersecting in $\Gamma = x+jy$ and operation of the six-port reflectometer depends on this intersection being unique, for which the centres must be neither coincident nor collinear. The centres can be found, for dropping the suffix k, equation (2.13) can be written as

$$|w|^2(c\Gamma+1)(c^*\Gamma^*+1) = (d\Gamma+e)(d^*\Gamma^*+e^*) \hspace{2cm} (2.14)$$

where * denotes complex conjugate.

$$\therefore \quad |w|^2(|c|^2|\Gamma|^2+c\Gamma+c^*\Gamma^*+1) = |d|^2|\Gamma|^2+de^*\Gamma+d^*e\Gamma^*+|e|^2$$

$$\therefore \quad |\Gamma|^2 - \frac{c|w|^2-d\,e^*}{|d|^2-|c|^2|w|^2}\Gamma - \frac{c^*|w|^2-d^*e}{|d|^2-|c|^2|w|^2}\Gamma^* = \frac{|w|^2-|e|^2}{|d|^2-|c|^2|w|^2}$$

Expressing this in the form $|\Gamma|^2-\Gamma_0^*\Gamma-\Gamma_0\Gamma^* = r^2-|\Gamma_0|^2$, where r is positive real, identifies it as the circle $|\Gamma-\Gamma_0|^2 = r^2$. Its centre is

$$\Gamma_0 = \frac{c^*|w|^2-d^*e}{|d|^2-|c|^2|w|^2} = \frac{(c_R|w|^2-(d_Re_R+d_Ie_I))+j((d_Ie_R-d_Re_I)-c_I|w|^2)}{|d|^2-|c|^2|w|^2}$$

Since $|\Gamma_0|^2 = \Gamma_0\Gamma_0^*$, it follows from the above that

$$(|d|^2-|c|^2|w|^2)^2|\Gamma_0|^2 = (c|w|^2-de^*)(c^*|w|^2-d^*e)$$

$$= |d|^2|e|^2+|c|^2|w|^4-(d^*ce+dc^*e^*)|w|^2$$

and

$$(|d|^2-|c|^2|w|^2)^2(r^2-|\Gamma_0|^2) = (|w|^2-|e|^2)(|d|^2-|c|^2|w|^2)$$

$$= -|d|^2|e|^2-|c|^2|w|^4+(|d|^2+|c|^2|e|^2)|w|^2$$

Adding gives

$$r^2 = \frac{|w|^2(|d|^2+|c|^2|e|^2-d^*ce-dc^*e^*)}{(|d|^2-|c|^2|w|^2)^2} = \frac{|w|^2|d-ce|^2}{(|d|^2-|c|^2|w|^2)^2}$$

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Thus for each k, $|w_k|$ generates the circle

$$\left(x - \frac{c_R|w_k|^2-(d_Re_R+d_Ie_I)_k}{|d_k|^2-|c|^2|w_k|^2}\right)^2 + \left(y - \frac{(d_Ie_R-d_Re_I)_k-c_I|w_k|^2}{|d_k|^2-|c|^2|w_k|^2}\right)^2 = \frac{|d_k-ce_k|^2|w_k|^2}{(|d_k|^2-|c|^2|w_k|^2)^2}$$

$$(2.15)$$

Now the assumption of unequal transmission between all ports implies that d_k and e_k are different for all k and equation (2.15) shows that the circle centres are therefore neither coincident nor collinear. Generally both the centre and radius of each circle is dependent on $|w|$ but if $c = 0$ then the radius of each circle is simply $r_k = |w/d|_k$ and the centre co-ordinates $(\Gamma_o)_k$ are independent of $|w|$. It follows from (2.12) and (2.13) that $c = 0$ implies that detector D_3 is isolated from the wave a_2 reflected onto the measurement port so that equation (2.13) becomes a linear relation between $|w_k|$ and Γ; this has been utilised for one form of five-port reflectometer (11).

In general then, $\Gamma = x+jy$ is uniquely determined from the three circles each dependent on the power ratios $|w_k|^2$ and can be found if sufficient can first be found of the seven complex numbers c, d_k, e_k. One scheme for calibrating a six-port for measuring VRC has been described by Woods (12). It requires the connection of seven standards providing different known Γ_ℓ $(1 \leq \ell \leq 7)$ and observing the twenty-one corresponding $|w_k|_\ell^2$ but its theory is particularly simple to derive from equation (2.14). Noting that

$$c\Gamma + c^*\Gamma^* = 2(c_Rx - c_Iy)$$

and

$$d\Gamma e^* + d^*\Gamma^*e = 2(d_Re_R+d_Ie_I)x - 2(d_Ie_R-d_Re_I)y$$

gives (2.14) in the form

$$|w|^2(1+|c|^2|\Gamma|^2+2(c_Rx-c_Iy)) = |d|^2|\Gamma|^2+|e|^2+2(d_Re_R+d_Ie_I)x-2(d_Ie_R-d_Re_I)y$$

Reintroducing suffix k and writing

$$p_k = (d_Re_R+d_Ie_I)_k \quad \text{and} \quad q_k = -(d_Ie_R-d_Re_I)$$

gives

$$|w_k|_\ell^2(1+|c|^2(x_\ell^2+y_\ell^2)+2(c_Rx_\ell-c_Iy_\ell)) = |d_k|^2(x_\ell^2+y_\ell^2)+|e_k|^2+2p_kx_\ell+2q_ky_\ell$$

$$(2.16)$$

Observing $|w_k|_\ell^2$ gives for each k an exact set of seven simultaneous linear equations (2.16) in real unknowns to enable $|c|^2$, c_R, c_I, $|d_k|^2$, $|e_k|^2$, p_k, q_k to be found. If more than seven standards are available then the over-determined set of equation (2.16) can be solved by standard methods (see section 6). The solutions can then be used in equation (2.16) recast as

$$(x^2+y^2)(|c|^2|w_k|^2-|d_k|^2)+2(c_R|w_k|^2-p_k)x-2(c_I|w_k|^2+q_k)y = |e_k|^2-|w_k|^2$$

to find $\Gamma = x+jy$ from the corresponding observed $|w_k|^2$. Writing

$$f_k = \frac{p_k - c_R|w_k|^2}{|c|^2|w_k|^2-|d_k|^2} \ , \quad g_k = \frac{q_k + c_I|w_k|^2}{|c|^2|w_k|^2-|d_k|^2} \ ,$$

$$r_k^2 = \frac{|e_k|^2-|w_k|^2}{|c|^2|w_k|^2-|d_k|^2} + f_k^2 + g_k^2$$

finally gives

$$(x-f_k)^2 + (y-g_k)^2 = r_k^2$$

which can be solved by writing

$$E = \frac{r_1^2-r_2^2+(f_2^2+g_2^2)-(f_1^2+g_1^2)}{2(f_2-f_1)} \ , \quad F = \frac{r_2^2-r_3^2+(f_3^2+g_3^2)-(f_2^2+g_2^2)}{2(f_3-f_2)}$$

giving

$$y = (F-E)/\left(\frac{g_3-g_2}{f_3-f_2} - \frac{g_2-g_1}{f_2-f_1}\right) \text{ and } x = E - \left(\frac{g_2-g_1}{f_2-f_1}\right)y$$

Other methods of calibration have been described which rely on the use of only four standards namely four short- or open-circuits (13). We proceed in the next section to derive a modification of the method derived in section 2.2 for a four-port to enable a six-port reflectometer to be calibrated by means of only three precisely-known and one approximately-known standard. This will pave the way to treating the dual six-port network analyser and its calibration in terms of just one known standard.

3 SIX- AND FOUR-PORT REFLECTOMETER EQUIVALENCE

3.1 SIX- TO FOUR-PORT REDUCTION THEORY

The six- to four-port reduction algorithm treated in this section is based on a derivation given by Engen (6). It enables a vector "indication" to be calculated from the power ratios observed for a six-port equivalent to that given by the receiver of a four-port reflectometer.

We define, consistent with (2.13), $w_k = (d_k\Gamma+e_k)/(c\Gamma+1)$ and solving for Γ gives

$$\Gamma = \frac{e_1-w_1}{cw_1-d_1} = \frac{e_2-w_2}{cw_2-d_2} = \frac{e_3-w_3}{cw_3-d_3}$$

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Writing $Q_k = |w_k|^2$ and eliminating Γ produces the three equations

$$Q_1 = |w_1|^2 \quad ; \quad A^2 Q_2 = |w_1-m|^2 \quad ; \quad B^2 Q_3 = |w_1-n|^2 \qquad (3.1)$$

with

$$A = \left|\frac{ce_1-d_1}{ce_2-d_2}\right| \quad ; \quad B = \left|\frac{ce_1-d_1}{ce_3-d_3}\right| \quad ; \quad m = \frac{d_1e_2-d_2e_1}{ce_2-d_2} \quad ; \quad n = \frac{d_1e_3-d_3e_1}{ce_3-d_3}$$

These three equations in w_1 represent three circles in the complex $w_1 = u_1 + jv_1$ plane intersecting in w_1. Their intersection is unique to each Γ because the three centres $(0,m,n)$ are bilinear transformations of those of (2.14). Now Q_k are the observed power ratios and since eliminating Γ has shown that A and B are constant for all Γ, it follows that they are characteristic of the reflectometer. They represent, in the w_1 plane, ratios of the radii of the two circles centred at (m,n) to that of the circle centred at the origin. We proceed to define three more real constants characterising the reflectometer, namely the distances between the centres of these three circles in the complex w_1 plane.

Writing $m = M\cos\mu + jM\sin\mu$, $n = N\cos\nu + jN\sin\nu$, $w_1 = u_1 + jv_1$ (M,N > 0) into (3.1) gives

$$\left.\begin{aligned}
Q_1 &= u_1^2 + v_1^2 \\
A^2 Q_2 &= (u_1-M\cos\mu)^2 + (v_1-M\sin\mu)^2 \\
B^2 Q_3 &= (u_1-N\cos\nu)^2 + (v_1-N\sin\nu)^2
\end{aligned}\right\} \qquad (3.2)$$

Now define

$$\begin{aligned}
p &\equiv |m-n|^2 = M^2 + N^2 - 2MN\cos(\nu-\mu) \quad ; \\
q &\equiv |n|^2 = N^2 \quad ; \\
r &\equiv |m|^2 = M^2 \quad .
\end{aligned} \qquad (3.3)$$

These definitions of m,n associated with equations (3.1) are independent of Γ so that these distances between the circle centres in the w_1 plane (p,q,r) also characterise the reflectometer (the p and q so defined are not related to those of (2.10) or (2.16)).

All five positive real numbers A^2, B^2, p, q, r can be found by observing the power ratios corresponding to nine (or more) DUT whose VRC are known only to be different; their precise values need not be known. We write from equations (3.2) and (3.3)

$$A^2 Q_2 - Q_1 = -2Mu_1\cos\mu - 2Mv_1\sin\mu + M^2 \quad \text{and} \quad B^2 Q_3 - Q_1 = -2Nu_1\cos\nu - 2Nv_1\sin\nu + N^2$$

or

$$\begin{bmatrix} \cos\mu & \sin\mu \\ \cos\nu & \sin\nu \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} (M^2+Q_1-A^2Q_2)/(2M) \\ (N^2+Q_1-B^2Q_3)/(2N) \end{bmatrix} \qquad (3.4)$$

Since $\sin(\nu-\mu) = \sin\nu\cos\mu - \cos\nu\sin\mu$, we have

$$\begin{bmatrix} \cos\mu & \sin\mu \\ \cos\nu & \sin\nu \end{bmatrix}^{-1} = (\sin(\nu-\mu))^{-1} \begin{bmatrix} \sin\nu & -\sin\mu \\ -\cos\nu & \cos\mu \end{bmatrix}$$

so the solution of (3.4) is

$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = (\sin(\nu-\mu))^{-1} \begin{bmatrix} \sin\nu & -\sin\mu \\ -\cos\nu & \cos\mu \end{bmatrix} \begin{bmatrix} (M^2+Q_1-A^2Q_2)/(2M) \\ (N^2+Q_1-B^2Q_3)/(2N) \end{bmatrix} \qquad (3.5)$$

which is always valid since $\sin(\nu-\mu) \neq 0$ because the circle centres are not collinear (this follows the definitions of ν and μ).

Now define

$$\alpha = (p-q-r)/(2\sqrt{qr}) \ ; \quad \beta = (r+Q_1-A^2Q_2)/(2\sqrt{r}) \ ;$$
$$\gamma = (q+Q_1-B^2Q_3)/(2\sqrt{q}) \ ; \qquad (3.6)$$

using positive square roots in the denominators and noting that p,q,r are necessarily positive. Then from (3.3)

$$\left. \begin{aligned} \alpha &= -\cos(\nu-\mu) = -\cos\nu\cos\mu - \sin\nu\sin\mu \\[1em] \beta &= (M^2+Q_1 - A^2Q_2)/(2M) \\[1em] \gamma &= (N^2+Q_1 - B^2Q_3)/(2N) \end{aligned} \right\} \qquad (3.7)$$

Using (3.7), (3.5) becomes

$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = (\sin(\nu-\mu))^{-1} \begin{bmatrix} \sin\nu & -\sin\mu \\ -\cos\nu & \cos\mu \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \end{bmatrix} \qquad (3.8)$$

But from this and (3.2) we have

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$$Q_1 = u_1^2 + v_1^2 = \begin{bmatrix} u_1 \\ v_1 \end{bmatrix}^T \begin{bmatrix} u_1 \\ v_1 \end{bmatrix}$$ (where T denotes the matrix transpose)

$$= \begin{bmatrix} \beta \\ \gamma \end{bmatrix}^T \begin{bmatrix} \sin\nu & -\cos\nu \\ -\sin\mu & \cos\mu \end{bmatrix} (\sin(\nu-\mu))^{-2} \begin{bmatrix} \sin\nu & -\sin\mu \\ -\cos\nu & \cos\mu \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \end{bmatrix}$$

$$= \begin{bmatrix} \beta \\ \gamma \end{bmatrix}^T (\sin(\nu-\mu))^{-2} \begin{bmatrix} 1 & \alpha \\ \alpha & 1 \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \end{bmatrix}$$ (using (3.7))

$$= (\sin(\nu-\mu))^{-2} (\beta^2 + \gamma^2 + 2\alpha\beta\gamma)$$

$$= (\beta^2 + \gamma^2 + 2\alpha\beta\gamma)/(1-\alpha^2)$$ (using (3.7) again)

Substituting for α, β, γ from (3.6) and simplifying gives

$$pQ_1^2 + qA^4Q_2^2 + rB^4Q_3^2 + (r-p-q)A^2Q_1Q_2 + (q-p-r)B^2Q_1Q_3 + (p-q-r)A^2B^2Q_2Q_3 \tag{3.9}$$
$$+ p(p-q-r)Q_1 + q(q-p-r)A^2Q_2 + r(r-p-q)B^2Q_3 + pqr = 0$$

Connecting at least nine loads known only to have different VRC and observing the power ratios Q_k corresponding to each provides (on dividing (3.9) throughout by (pqr)) at least nine simultaneous inhomogeneous linear equations in the nine unknown coefficients

$$X_1 = p/(pqr) = 1/(qr) \qquad\qquad X_4 = A^2(r-p-q)/(pqr)$$

$$X_2 = qA^4/(pqr) = A^4/(pr) \qquad X_5 = B^2(q-p-r)/(pqr)$$

$$X_3 = rB^4/(pqr) = B^4/(pq) \qquad X_6 = A^2B^2(p-q-r)/(pqr)$$

$$X_7 = (p(p-q-r))/(pqr) = (p-q-r)/(qr) \tag{3.10}$$

$$X_8 = (qA^2(q-p-r))/(pqr) = A^2(q-p-r)/(pr)$$

$$X_9 = (rB^2(r-p-q))/(pqr) = B^2(r-p-q)/(pq)$$

where, from (3.9) and (3.10)

$$X_1Q_1^2 + X_2Q_2^2 + X_3Q_3^2 + X_4Q_1Q_2 + X_5Q_1Q_3 + X_6Q_2Q_3 + X_7Q_1 + X_8Q_2 + X_9Q_3 = -1$$

$$\tag{3.11}$$

The set of equations (3.11) may be solved by a standard least-squares method (see section 6) and from equation (3.10) can be deduced the relations

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$$r = \frac{2X_5 - X_7X_9}{2X_1X_9 - X_5X_7} \; ; \qquad q = \frac{2X_4 - X_7X_8}{2X_1X_8 - X_4X_7} \; ;$$

$$p = r+q+(X_7/X_1) \; ; \quad A^2 = +\sqrt{prX_2} \; ; \quad B^2 = +\sqrt{pqX_3} \tag{3.12}$$

in which use has been made of A^2 and B^2 being positive.

Equations (3.11) and (3.12) enable approximations to the five required real numbers to be found. The solutions are likely to be only approximate because, as Engen remarks (6), they represent conditions of tangency of three planes to the ellipsoidal paraboloid represented by equation (3.9) which is a quartic in $|w_k|$. The solutions are refined by using a standard Gauss-Newton iterative solution for the five unknowns of the nine (or more) equations (3.9). (We remark that since these five reals characterise the reflectometer and are found by using at least nine DUT known only to have different VRC, they provide a means of monitoring the long-term stability of the instrument with loads requiring stability only during observations of the corresponding power ratios Q_k.)

The purpose of finding these five reals p,q,r,A^2,B^2 is to enable a unique vector w to be calculated from the three Q_k observed for any subsequently connected load, of course. This can be done by writing

$$w \equiv u+jv \equiv w_1(\cos\mu - j\sin\mu) = (u_1+jv_1)(\cos\mu - j\sin\mu)$$

or

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \cos\mu & \sin\mu \\ -\sin\mu & \cos\mu \end{bmatrix}\begin{bmatrix} u_1 \\ v_1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\mu & \sin\mu \\ -\sin\mu & \cos\mu \end{bmatrix}(\sin(\nu-\mu))^{-1}\begin{bmatrix} \sin\nu & -\sin\mu \\ -\cos\nu & \cos\mu \end{bmatrix}\begin{bmatrix} \beta \\ \gamma \end{bmatrix} \quad \text{from (3.8)}$$

$$= \begin{bmatrix} 1 & 0 \\ -\cot(\nu-\mu) & \csc(\nu-\mu) \end{bmatrix}\begin{bmatrix} \beta \\ \gamma \end{bmatrix}$$

or from (3.7)

$$u = \beta \; , \quad v = (\alpha\beta+\gamma)/\pm\sqrt{1-\alpha^2} \tag{3.13}$$

But from the definition of w_k we have

$$w = w_1(\cos\mu - j\sin\mu) = (d\Gamma+e)/(c\Gamma+1)$$

which is identical to equation (2.7) derived for a four-port reflectometer in which d and e are new constants of the six-port (given by $d = d_1(\cos\mu - j\sin\mu)$ and $e = e_1(\cos\mu - j\sin\mu)$). Since α,β,γ are known from (3.6) for any Q_k, we can find the vector w corresponding to any load VRC, apart from the sign ambiguity in (3.13). This sign ambiguity in the six-port reflectometer arises from that of the sign of $\sin(\nu-\mu)$ and is equivalent

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to not knowing in advance which of the two scalar indications of the receiver
(or vector voltmeter) of a four-port reflectometer represents u and which
represents v. A way of resolving this by a slight modification of the four-
port calibration method of section 2.2, and a practical implementation of a
suitable six-port calibration scheme, are both treated in the next section.

3.2 SIX-PORT CALIBRATION USING "3½" STANDARDS

With the aim of reducing the number of sources of uncertainty of
measurement by minimising the number of precisely-known standards needed,
we can employ the algorithm of section 3.1 to calibrate a six-port reflecto-
meter with three such standards plus a load whose VRC is only approximately
known (to resolve the ambiguity referred to). The power ratios Q_k (k = 1,2,3)
are observed when these four loads having VRC of $\Gamma, \Gamma', \Gamma'', \Gamma'''$ are connected in
turn and the corresponding w, w', w'', w''' are calculated using the same sign
for each in equation (3.13). This determines either the four w or their
complex conjugates w^*. Now it can be verified by substitution in equation
(2.7) (or deduced from the theory of cross-ratio groups) that

$$\frac{(w-w''')\,(w'-w'')}{(w-w')\,(w''-w''')} = \frac{(\Gamma-\Gamma''')\,(\Gamma'-\Gamma'')}{(\Gamma-\Gamma')\,(\Gamma''-\Gamma''')} \tag{3.16}$$

But the sign of the imaginary part of the RHS of equation (3.16) is known
from our knowledge of the four VRC and if it disagrees with that of the LHS
then the sign of v selected in equation (3.13) must be changed. The method
will fail if all four VRC have the same magnitude because the RHS of
equation (3.16) would then be real, but we describe below a calibration
method used at RSRE to avoid this failure. It also avoids the apparent
need to make at least nine physical connections to the reflectometer to
provide data for the six- to four-port reduction algorithm.

This procedure has been employed to calibrate different six-port
reflectometers at wavelengths ranging from metres to millimetres and it
relies on a short- or open-circuit uncalibrated variable attenuator to
provide six or more different VRC for use with the algorithm. Power ratios
Q_k are observed for each of the following:

    a. Three known standards connected in turn to provide three precisely
    known different VRC. (In millimetric waveguide these have been an
    electroformed short-circuit first connected directly and then via two
    known lengths of precision waveguide (spacers) in succession. At VHF
    a short-circuit and two calculable coaxial capacitors have been used.)

    b. A nominally matched load.

    c. A variable attenuator set to at least three settings first when
    unterminated and then when terminated with a short-circuit. (This
    provides two sets of three different VRC for which the phase of the
    two sets differs by more than $90°$.)

This provides a set of ten loads known to have different VRC and the
corresponding power ratios are used in the algorithm of section 3.1 to
calculate A^2, B^2, p, q, r. Thus equations (3.9),(3.11),(3.12) are used in
sequence to yield approximate values and then successively better approxi-
mations are found by applying the Gauss-Newton technique (outlined in
section 6) to equation (3.9). The power ratios corresponding to the first

four loads (steps a and b above) are then used in equations (3.6) and (3.13) to calculate the LHS of equation (3.16), the known values of Γ being used in the RHS to resolve the ambiguity of sign of v in (3.13). The three precisely known Γ and the corresponding values of w are then used in equation (2.9) to complete the calibration of the equivalent four-port. Thereafter, the VRC of any subsequently connected load is found by applying equations (3.6), (3.13) and (2.10) in sequence.

In practice, the nominally matched load at (b) in this procedure can be provided by setting the attenuator to provide its maximum attenuation, so that the procedure involves making only four physical connections to the measurement port. We have found it necessary to use the set of ten VRC known to be different (rather than the minimum of nine) to ensure reliable convergence of the Gauss-Newton routine used in solving equation (3.9). We have found it sufficient to terminate the process of successive approximation of the five real numbers A^2, B^2, p, q, r when a precision of 1 in 10^6 has been reached for each.

For a well-designed six-port reflectometer junction, the squared distances p, q, r between the circle centres in the w_1 plane are all of the same order. The reliability of convergence obtained with the procedure described has been tested at RSRE by employing it successfully for a six-port junction maladjusted to the extent that p was two orders less in magnitude than q or r.

As remarked earlier, the five real numbers found by this algorithm provide a means of monitoring the stability of the instrument (since their values do not depend on known Γ). We also remark that the redundancy provided by using the algorithm minimises the variance of, in effect, the w from their best fit values; this tends to average out the effects of any departure of detector response from linear power operation. This means that the values of Γ which can be measured for loads used in the process (after reduction and calibration have been completed) may be more precise than those obtained by subsequent measurements.

3.3 CALIBRATION IN TERMS OF A COMMON Z_o

We have so far made the assumption in deriving calibration procedures that there is no difficulty in realising a set of practical calibration standards to provide at least three precisely-known different VRC Γ_ℓ normalised to the same Z_o. For example, in calibrating six-ports with a waveguide short-circuit and two known lengths of guide we have approximated to truth by considering the cross-sections of the waveguide to be identical and the standards to be lossless. The precision of measurement achieved at RSRE with six-port reflectometers is greater than that of these approximations for commercial-quality waveguide and the performance recently reported of an improved movable short-circuit (18) suggests that it is worthwhile removing at least the need to assume lossless waveguide (although the VRC of the short-circuit must be assumed known). This can be done by applying to a six-port a method developed by da Silva and McPhun of calibrating a four-port reflectometer (14).

The method assumes that the VRC of a reflector (the short-circuit) retains a constant value Γ_o irrespective of its position along a uniform transmission line or waveguide. If the power ratios Q_k are observed at scale settings of $\ell_o = 0$, $(\ell_o + \ell)$, $(\ell_o + 2\ell)$, $(\ell_o + 3\ell)$ along the line measured

from the connection, then the VRC at the connection will be Γ_0,
$\Gamma' = \Gamma_0 \exp(-2\delta\ell)$, $\Gamma'' = \Gamma_0 \exp(-4\delta\ell)$, $\Gamma''' = \Gamma_0 \exp(-6\delta\ell)$, δ being the propagation constant of the line. Corresponding vector indications w_0, w', w'', w''' can be calculated from the power ratios and, from equation (3.16)

$$\frac{(w_0 - w''')(w' - w'')}{(w_0 - w')(w'' - w''')} = \frac{(\Gamma_0 - \Gamma''')(\Gamma' - \Gamma'')}{(\Gamma_0 - \Gamma')(\Gamma'' - \Gamma''')} \tag{3.17}$$

Writing $x = \exp(-2\delta\ell)$ then the RHS of this equation is $(x^2+x+1)/x$ and denoting the LHS (calculated using the positive root in (3.13)) by W, then

$$W = (x^2+x+1)/x \quad \text{and so} \quad x = (W-1 \pm \sqrt{(W-1)^2-4})/2 \tag{3.18}$$

The two roots of (3.18) are reciprocals of each other and only the one with modulus less than unity is admissible, since the transmission line attenuates. The other possible value of the LHS of (3.17) is the complex conjugate W^* and the version of (3.18) corresponding to this has the admissible root x^*, where x is the admissible root obtained from W. To choose between x and x^* we remember that theory predicts that for smooth conductors $\delta \triangleq (jk)$, where k is the free-medium angular wave number in the transmission line (usually written, with symbol usage different to that of this paper, ω/v or $\omega\sqrt{\epsilon\mu}/c$). Let Log represent the complex logarithm function with phase θ mapped into the range $-\pi \leqslant \theta < \pi$, then if x, rather than x^*, is correct we must have

$$\text{Log } x \triangleq -2jk\ell + 2n\pi \tag{3.19}$$

where n is a suitable integer.

In particular the signs of the imaginary parts of the LHS and RHS of (3.17) will agree. If they do, we were correct in choosing x, W and the positive sign in (3.13); if not then we should have chosen x^*, W^* and the negative sign in (3.13). This is sufficient to resolve the ambiguity, for the sign of (3.13) is independent of Γ; henceforth we shall denote the correct root by x.

We now know x, so

$$\Gamma_0, \quad \Gamma' = \Gamma_0 x, \quad \Gamma'' = \Gamma_0 x^2, \quad \Gamma''' = \Gamma_0 x^3$$

are all known multiples of Γ_0, and we know the corresponding w_0, w', w'', w''' so that, from equation (2.7)

$$d\Gamma_0 + e - w_0 c\Gamma_0 = w_0 \quad ; \quad d\Gamma_0 x + e - w' c\Gamma_0 x = w' \quad ;$$

$$d\Gamma_0 x^2 + e - w'' c\Gamma_0 x^2 = w'' \quad ; \quad d\Gamma_0 x^3 + e - w''' c\Gamma_0 x^3 = w'''$$

which are four simultaneous linear equations in the three unknowns $(c\Gamma_0)$, $(d\Gamma_0)$, e which can be solved by a standard least-squares method. This knowledge allows the ratio of any two VRC Γ, Γ' to be calculated from the corresponding indications w, w' obtained from the observed power ratios for, from equation (2.7),

$$\Gamma = \frac{w - e}{d - wc}$$

$$\therefore \quad \frac{\Gamma}{\Gamma_o} = \frac{w - e}{(d\Gamma_o) - w(c\Gamma_o)}$$

But $\quad \Gamma/\Gamma' = (\Gamma/\Gamma_o)/(\Gamma'/\Gamma_o)$

so that the ratio can be found. To obtain any Γ absolutely, however, Γ_o must be known.

Variations of results obtained in experiments at RSRE using a sliding short-circuit to implement this calibration method using 7 mm bore precision 50 ohm airline as a standard showed that the assumption of a constant $\Gamma_o = -1$ for all short-circuit positions was inadequate. Nevertheless, apart from its possible application were improved short-circuits available, it is of interest because of its affinity with the method of calibrating a dual six-port network analyser described later. This uses as the standard a known length ℓ of uniform line (or waveguide); a short-circuit which is assumed only to have the same VRC for each connection is also used but its Γ_o is measured by the calibration process, as is the $x = \exp(-2\delta\ell)$ of the line.

3.4 POWER FLUX FROM THE MEASUREMENT PORT

So far we have assumed that each of the four detectors provides an indication only proportional to the power absorbed by it. If now we have a power <u>meter</u> (ie one which correctly indicates the power absorbed) to serve as a standard of power then a six-port reflectometer already calibrated for VRC measurement can be calibrated to indicate the power flux emerging from its measurement port. It may then be used for comparing power meters by connecting them in turn, eg for comparing the effective efficiencies of bolometer mounts.

From equation (2.12)

$$P_3 = |\alpha_3 \Gamma + \beta_3|^2 |b_2|^2$$

$$= |\beta_3|^2 |1 + c\Gamma|^2 |b_2|^2$$

If the power absorbed by the standard power meter having VRC Γ_L is P_L then

$$P_L = G_o(|b_2|^2 - |a_2|^2)$$

$$= G_o(1 - |\Gamma_L|^2)|b_2|^2 \qquad (3.20)$$

where G_o is the conductive component of the admittance characterising the mode transmitted at the measurement port.

By division

$$P_L = \frac{G_o}{|\beta_3|^2} \frac{(1-|\Gamma_L|^2)}{|1+c\Gamma_L|^2} P_3 \qquad (3.21)$$

Substituting for Γ_L from equation (2.7)

$$P_L = \frac{G_o}{|\beta_3|^2} \frac{(|d-wc|^2 - |w-e|^2)}{|d-ce|^2} P_3$$

Writing

$$K = \frac{G_o}{|\beta_3|^2 |d-ce|^2}$$

as a constant characterising the reflectometer gives

$$P_L = K(|d-wc|^2 - |w-e|^2)P_3 \qquad (3.22)$$

Both K in (3.22) or $G_o/|\beta_3|^2$ in (3.21) characterise the reflectometer and can be found by observing Q_k and the indication of the standard power meter when connected to the measurement port; thereafter the power absorbed by any DUT can be found from either of equations (3.21) or (3.22).

4   THE DUAL REFLECTOMETER NETWORK ANALYSER

4.1   PRINCIPLES, CALIBRATION PROCEDURE AND NOTATION

So far we have been concerned only with measuring the VRC and power absorbed by one-port DUTs, but the scattering coefficients of two-port junctions can be measured with two reflectometers connected to the same source and a   phase-changer, as shown in Figure 3. We illustrate the principle of this by assuming that the reflectometers have been calibrated so that their indications are apparent VRCs in which the "reflected" waves comprise both those reflected by and transmitted through the two-port junction under test. Thus, using the notation of Figure 3, we have $\Gamma_A = a_A/b_A$ and $\Gamma_B = a_B/b_B$. Denoting the scattering coefficients of the DUT by s_{ij} in the usual way gives

$$\left.\begin{array}{l} a_A = s_{11}b_A + s_{12}b_B \\ a_B = s_{21}b_A + s_{22}b_B \end{array}\right\} \qquad (4.1)$$

so that $\Gamma_A = s_{11} + s_{12} \dfrac{b_B}{b_A}$ and $\Gamma_B = s_{21} \dfrac{b_A}{b_B} + s_{22}$

Eliminating (b_A/b_B) from these two equations gives

$$\Gamma_B s_{11} + \Gamma_A s_{22} + (s_{12}s_{21}-s_{11}s_{22}) = \Gamma_A\Gamma_B \qquad (4.2)$$
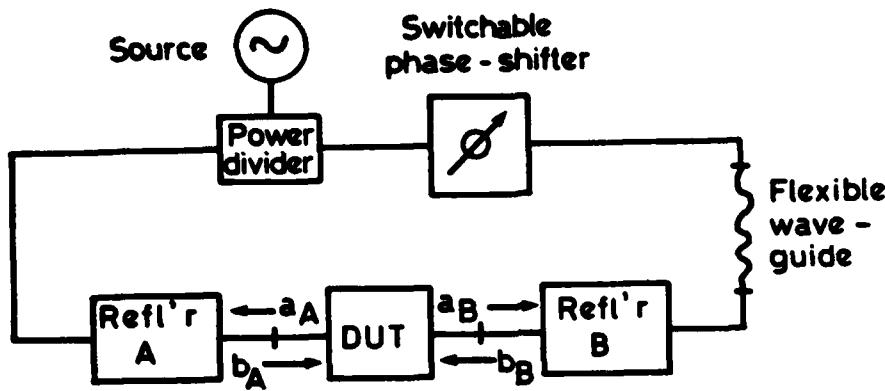
FIG.3. DUAL - REFLECTOMETER NETWORK ANALYSER

By setting the adjustable phase-shifter to three (or more) values known only to provide different phase relations between b_A and b_B, there are obtained three (or more) sets of $(\Gamma_A)_i$, $(\Gamma_B)_i$, $(i = 1,2,3 ..)$ which result in a set of three (or more) simultaneous equations (4.2) which can be solved for s_{11}, s_{22} and the product $s_{12}s_{21}$. This allows determination of the scattering coefficients of any two-port junction that is reciprocal (in the sense that $s_{12} = s_{21}$ when both ports transmit the same mode (15)), provided we ignore for the moment the ambiguity of phase of s_{12}. We remark that the phase-shifter needs constancy only during each measurement and does not need to accurately repeat its settings for a sequence of measurements (unless, as we shall see, there is a requirement to measure phase angles $\angle s_{12}$ and $\angle s_{21}$ of a non-reciprocal two-port junction). We also remark that the reflectometers need not be six-ports and that a phase-shifter with four settings allows the measured result to be the best fit to four sets of power ratio data (since it is a least-squares solution of an overdetermined set of equations (4.2)).

Each apparent VRC indicated during measurement of a passive, two-port junction may be greater than unity, as can be seen by considering the "null two-port junction" (ie when the two reflectometers are connected together) for then a_A and b_B are equal ($\Gamma_A = 1/\Gamma_B$). This implies that the reflecto-meter calibration methods so far derived may be inappropriate because they employ standards of known VRC of $|\Gamma| < 1$. Recalling the comments following equation (2.10), this means that either the excitation of each reflectometer must be halved after calibration to ensure that the detectors always operate in their linear range or it must be assumed that their response remains linear when $|\Gamma| > 1$. The calibration procedure we now consider is essentially the one described in (8) and utilises connection of both reflectometer measurement ports to each other and calibrates both instruments in terms of just one precisely known standard, namely a known length ℓ of precision waveguide or air-dielectric transmission line having the same uniform cross-

section as the measurement ports. The steps in the calibration procedure are illustrated in Figure 4 and they consist of observing the power ratios:

a. when the two measurement ports are connected together (condition 1);

b. when the two measurement ports are connected via the length ℓ of standard waveguide or coaxial line (condition 2);

c. when they are connected via a stable but unknown two-port, such as a 6 dB or 10 dB pad;

d. when there is zero transmission between the measurement ports (condition 3); this is achieved by terminating both ports with different unknown reflecting terminations, which are exchanged to allow two sets of observations to be made.
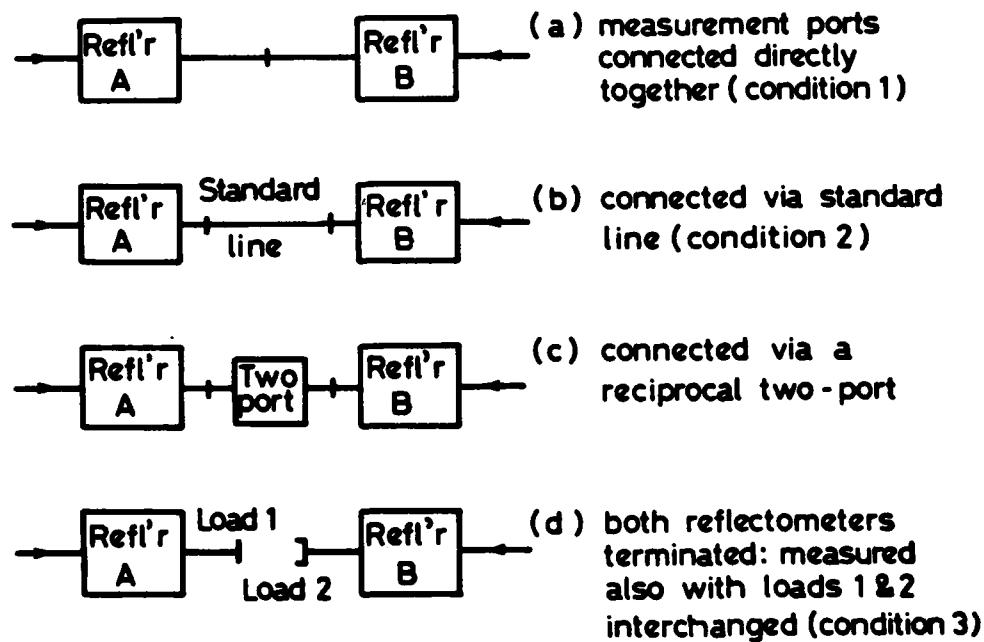
FIG.4. CALIBRATION PROCEDURE FOR DUAL-REFLECTOMETER
NETWORK ANALYSER

Only the three conditions numbered 1 to 3 in brackets are essential to the calibration process (except for measurement of the phase angles $\angle s_{12}$ and $\angle s_{21}$ of non-reciprocal two-port junctions). The object of using additional unknown one- and two-port devices is to provide overdetermined sets of equations to enhance the measurement precision. Every least-squares solution implies a consistency check and if the unknowns are stable they also provide checks of consistency of calibration.

All four states of phase excitation are used during steps a, b, c, but only one is needed when there is zero transmission at step d. There are therefore fourteen sets of observed power ratios for each six-port available as data for applying the algorithm of section 3.1 to reduce each to an equivalent four-port reflectometer (complete with its own ambiguity of sign of v resulting from equation (3.13)). These equivalent four-ports are modelled by the two-port error box representation of section 2.2 and the notation illustrated in Figure 5 is used with this representation. The waves incident on and emergent from the notional perfect reflectometers A and B are denoted by a_A, b_A and a_B, b_B; the vector indications of these notional reflectometers by $w_A = a_A/b_A$ and $w_B = a_B/b_B$; and Γ_A, Γ_B are used for the VRCs at the true measurement ports. With this notation, (4.1) can be applied to a two-port junction connected between the two notional measurement planes to relate w_A, w_B to its scattering coefficients (by analogy with the derivation of (4.2), giving

$$w_B s_{11} + w_A s_{22} + (s_{12}s_{21} - s_{11}s_{22}) = w_A w_B \qquad (4.3)$$

During measurement a DUT can be considered as being in cascade between the two error boxes so it is convenient to express (4.1) in a different matrix form

$$\begin{bmatrix} a_A \\ b_A \end{bmatrix} = \frac{1}{s_{21}} \begin{bmatrix} (s_{12}s_{21}-s_{11}s_{22}) & s_{11} \\ -s_{22} & 1 \end{bmatrix} \begin{bmatrix} b_B \\ a_B \end{bmatrix} \qquad (4.4)$$

The square matrix and its scalar multiplier in equation (4.4) is the so-called cascading matrix (15) and this has the important property that the cascading matrix of two (or more) two-port junctions connected in cascade is the ordinary matrix product of their individual cascading matrices.

We now need to introduce further notation (illustrated in Figure 5) to handle the observations made during conditions 1 to 3 of the calibration procedure.
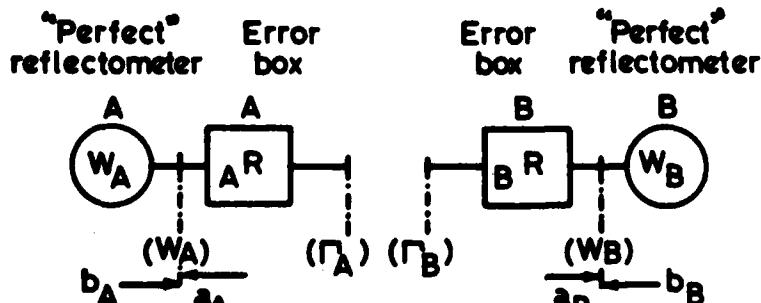
$_A R$ and $_B R$ are cascading matrices of error boxes

FIG.5. NOTATION USED FOR DUAL SIX-PORT NETWORK ANALYSER

Denoting the cascading matrices for error boxes A and B by $(_A R), (_B R)$, respectively, and that of the standard line by $(_L R)$, and using superscripts 1,2,3 to represent for these three conditions the vector indications w_A, w_B, enables us to write $^1 w_A$, $^1 w_B$ for these indications when the two reflecto-meters are connected together (ie connected via the "null two-port"), and $^2 w_A$, $^2 w_B$ when they are connected via the standard length of line, etc. This notation, plus the observation that the cascading matrix of the null two-port is just the (2x2) identity matrix and can therefore be suppressed, will enable us to set up cascading matrices to relate $^1 w_A$, $^1 w_B$, $^2 w_A$, $^2 w_B$ to $(_A R)$, $(_B R)$, $(_L R)$.

The two six-port reflectometers have eight power meters between them and these give seven independent power ratios, but the two sets of Q_k (as defined in section 3.1) account for only six of them. For measurements of non-reciprocal two-port junctions we also need the seventh and the defini-tions of the power ratios Q_k must be extended to include it. If the indi-cation from detector D_3 of Figure 1 is hereafter called P_R (instead of P_3) and the others renumbered 1,2,3 for each six-port then we have, in an obvious notation,

$$
\left.
\begin{array}{ll}
Q_{Ak} = P_{Ak}/P_{AR} & (k = 1,2,3) \\[2ex]
Q_{Bk} = P_{Bk}/P_{BR} & (k = 1,2,3) \\[2ex]
Q_{AB} = P_{BR}/P_{AR} &
\end{array}
\right\} \tag{4.5}
$$

Note that a measurement set of ratios includes only one Q_{AB}.

Having illustrated the principle of operation, described our calibra-tion procedure, and established our notation, we can proceed to derive our calibration theory. We tackle first the not inconsiderable problem of resolving the ambiguities of sign of v for the two equivalent four-port reflectometers.

4.2 RESOLVING SIGN AMBIGUITIES FOR DUAL SIX-PORT NETWORK ANALYSER

Having reduced each six-port to an equivalent four-port reflectometer, we apply equation (3.16) to the measurements made with condition 1 of the calibration procedure to give

$$
\frac{(^1 w_{1A} - ^1 w_{4A})(^1 w_{2A} - ^1 w_{3A})}{(^1 w_{1A} - ^1 w_{2A})(^1 w_{3A} - ^1 w_{4A})} = \frac{(^1 \Gamma_{1A} - ^1 \Gamma_{4A})(^1 \Gamma_{2A} - ^1 \Gamma_{3A})}{(^1 \Gamma_{1A} - ^1 \Gamma_{2A})(^1 \Gamma_{3A} - ^1 \Gamma_{4A})}
$$

$$
= \frac{(^1 \Gamma_{1B} - ^1 \Gamma_{4B})(^1 \Gamma_{2B} - ^1 \Gamma_{3B})}{(^1 \Gamma_{1B} - ^1 \Gamma_{2B})(^1 \Gamma_{3B} - ^1 \Gamma_{4B})}
$$

$$
= \frac{(^1 w_{1B} - ^1 w_{4B})(^1 w_{2B} - ^1 w_{3B})}{(^1 w_{1B} - ^1 w_{2B})(^1 w_{3B} - ^1 w_{4B})} \tag{4.6}
$$

because for the null two-port we have $^1\Gamma_{iA}{}^1\Gamma_{iB} = 1$ (i = 1,2,3,4) for all four phase states. The signs of the imaginary parts of the cross-ratios in w_A and w_B are compared and if they disagree then the negative sign is taken in equation (3.13) for either w_A or w_B (at this stage it does not matter which). This does not guarantee that both signs are now right but it does guarantee that either they are both right or both are wrong; that is, we are now using consistently for A and B either w or w* (but we do not yet know which). We assume for the moment that we have w and correct later if this is found to be wrong. It is worth noting that (4.6), unlike the use of (3.16) in section 3.2, is exact so that (after any necessary sign change) the two cross-ratios in w in (4.6) should be exactly equal. This is one of many consistency checks that this calibration method automatically provides.

Next, we calculate the elements of the cascading matrices of the two-ports between the notional measurement planes of w_A and w_B, first when the true measurement planes are connected together (condition 1) and then when they are connected via the length of standard transmission line (condition 2). To do this, we apply (4.3) for each of the four measurements (with different phase excitation) to each of the two conditions; for example for condition 1, this gives

$$^1w_{iB}{}^1s_{11} + {}^1w_{iA}{}^1s_{22} + ({}^1s_{12}{}^1s_{21} - {}^1s_{11}{}^1s_{22}) = {}^1w_{iA}{}^1w_{iB} \qquad (4.7)$$

$$(i = 1,2,3,4)$$

which provides four simultaneous inhomogeneous linear equations in the three unknowns $^1s_{11}$, $^1s_{22}$ and $({}^1s_{12}{}^1s_{21} - {}^1s_{11}{}^1s_{22})$; these unknowns may now be found by a standard method. From (4.4), we immediately have the elements of the corresponding cascading matrix, except for a common factor $(1/{}^1s_{21})$; this is ignored because (as we shall see) only the ratios of the elements are significant provided $^1s_{21} \neq 0$ (which is obviously true whenever transmission between the two notional planes of w_A and w_B is possible). Applying the same procedure to the measurements associated with condition 2 gives the elements of its corresponding cascading matrix, apart from an ignorable factor of $(1/{}^2s_{21})$. (We remark that, since the determinant of the elements of the cascading matrix of a reciprocal two-port junction with both ports referred to the same reference Z_o is unity, the elements can be determined absolutely, apart from a sign ambiguity; however, the assumption of reciprocity is an unnecessary restriction and yet another sign ambiguity is obviously undesirable.)

Denoting the cascading matrices just found for calibration conditions 1 and 2 by ^1R and ^2R, respectively, we see that they satisfy the equations

$$^1R = {}^1s_{21}(_AR)(_BR) \quad \text{and} \quad {}^2R = {}^2s_{21}(_AR)(_LR)(_BR) \qquad (4.8)$$

where the cascading matrix of the null two-port has been suppressed. We also have, using the notation of section 3.3 in (4.4)

$$_LR = \begin{bmatrix} \exp(-\delta\ell) & 0 \\ 0 & \exp(\delta\ell) \end{bmatrix} \qquad (4.9)$$

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Now define a matrix $T \equiv {}^2R({}^1R)^{-1} \det({}^1R)$. (As remarked earlier, $\det({}^1R)$ is usually unity but we do not need to assume that it is, only that it is $\neq 0$. For this, the sufficient condition is the physically essential relation ${}^1s_{21} \neq 0$.) The elements of T can be calculated from the known elements of ${}^1R$ and ${}^2R$; using ij to denote elements, we obtain

$$T_{ij} = {}^2R_{ij}\, {}^1R_{(3-j)(3-j)} - {}^2R_{i(3-j)}\, {}^1R_{(3-j)j} \qquad (4.10)$$

But we also have, on eliminating ${}_BR$ between the two equations (4.8),

$$^2R = {}^2s_{21}({}_AR)({}_LR)\left((1/{}^1s_{21})({}_AR)^{-1}\,{}^1R\right)$$

so

$$T = \det({}^1R){}^2R({}^1R)^{-1} = \det({}^1R)({}^2s_{21}/{}^1s_{21})({}_AR)({}_LR)({}_AR)^{-1}$$

or, on writing

$$\det({}^1R)({}^2s_{21}/{}^1s_{21})$$

as ϕ (a number, not a matrix),

$$T({}_AR) = \phi({}_AR)({}_LR)$$

Finally, we multiply this out, using the suffix notation and substituting from (4.9) for ${}_LR_{ij}$, to give

$$T_{11}\, {}_AR_{11} + T_{12}\, {}_AR_{21} = \phi\, {}_AR_{11} \exp(-\delta\ell) \qquad (4.11.1)$$

$$T_{21}\, {}_AR_{11} + T_{22}\, {}_AR_{21} = \phi\, {}_AR_{21} \exp(-\delta\ell) \qquad (4.11.2)$$

$$T_{11}\, {}_AR_{12} + T_{12}\, {}_AR_{22} = \phi\, {}_AR_{12} \exp(\delta\ell) \qquad (4.11.3)$$

$$T_{21}\, {}_AR_{12} + T_{22}\, {}_AR_{22} = \phi\, {}_AR_{22} \exp(\delta\ell) \qquad (4.11.4)$$

Dividing (4.11.1) by ${}_AR_{11}$ and (4.11.2) by ${}_AR_{21}$ gives

$$T_{11} + T_{12}({}_AR_{21}/{}_AR_{11}) = \phi \exp(-\delta\ell) = T_{21}({}_AR_{11}/{}_AR_{21}) + T_{22}$$

or

$$T_{21}({}_AR_{11}/{}_AR_{21})^2 + (T_{22}-T_{11})({}_AR_{11}/{}_AR_{21}) - T_{12} = 0 \qquad (4.12.1)$$

Treating (4.11.3) and (4.11.4) similarly gives

$$T_{21}({}_AR_{12}/{}_AR_{22})^2 + (T_{22}-T_{11})({}_AR_{12}/{}_AR_{22}) - T_{12} = 0 \qquad (4.12.2)$$

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Also, on dividing (4.11.1) by (4.11.4) and recalling the notation of section 3.3, we have

$$x = \exp(-2\delta\ell) = \frac{T_{11} + T_{12}(_A R_{21}/_A R_{11})}{T_{22} + T_{21}(_A R_{12}/_A R_{22})} \tag{4.13}$$

Now (4.12.1) and (4.12.2) show that $(_A R_{11})/(_A R_{21})$ and $(_A R_{12})/(_A R_{22})$ satisfy the same quadratic equation. There are two possibilities here; either the two element ratios are given by the same root of the quadratic, or one of them is equal to one root and the other to the other root. We can reject the first possibility because it requires the two element ratios to be equal which, from (4.4) requires that $_A S_{12}\ _A S_{21} = 0$; this is plainly non-physical. Taking the second possibility then, and solving (4.12) by formula, gives

$$(_A R_{12}/_A R_{22}),(_A R_{11}/_A R_{21}) = \frac{T_{11} - T_{22} \pm \sqrt{(T_{11} - T_{22})^2 + 4 T_{12} T_{21}}}{2 T_{21}} \tag{4.14}$$

Substituting these values into (4.13) gives

$$\exp(-2\delta\ell) = \frac{T_{11} + T_{22} \mp \sqrt{(T_{11} - T_{22})^2 + 4 T_{12} T_{21}}}{T_{11} + T_{22} \pm \sqrt{(T_{11} - T_{22})^2 + 4 T_{12} T_{21}}}$$

where the upper signs in the numerator and denominator correspond to taking the upper sign in (4.14) for $(_A R_{12})/(_A R_{22})$.

The two possible values are reciprocal so, as in section 3.3, we reject the one with modulus > 1. (This requires a precision of measurement that enables this discrimination to be effective for the few centimetres of air dielectric line used as the calibration standard. It has been found to be effective for 75 mm of 7 mm bore precision coaxial line giving about 0.004 dB attenuation; alternative methods similar to that of section 3.2 are available, however, for systems with insufficient precision.) Now, consider the effect of choosing the wrong sign in (4.6), so that w^* has been used throughout instead of w. Reviewing the derivation so far, it follows that the effect would be to give $^1 s^*$ and $^2 s^*$ for $^1 s$ and $^2 s$, and similarly $^1 R^*$, $^2 R^*$, T^*, $(_A R_{12}/_A R_{22})^*$, $(_A R_{11}/_A R_{21})^*$ and x^* instead of their unconjugated forms. We resolve this by using the method of section 3.3 to choose between x and x^* and all other quantities are automatically resolved as well.

It is important to note, however, that with δ approximately equal to jk, the discrimination depends on the ability to distinguish between $\exp(-2jk\ell)$ and $(\exp(-2jk\ell))^* = \exp(2jk\ell)$. If these quantities are essentially equal, as will happen if the length ℓ is too close to an integral number of quarter-wavelengths, then the method will not work. The determined difference between δ and jk affords a very sensitive test of the repeatability of calibration.

All the ambiguous signs having been sorted out for the two equivalent four-port reflectometers, we can now derive the theory for their calibration.

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4.3 DUAL REFLECTOMETER CALIBRATION THEORY

We begin by applying equation (4.3) to transform from w_A to Γ_A instead of from w_A to w_B. Prefixing s_{ij} by A to represent the elements of the scattering matrix of error box A we have, remembering that we must invert Γ_A to allow for the reversed measurement direction (as shown by the arrows in Figure 3).

$$\Gamma_A^{-1}{}_A s_{11} + w_A {}_A s_{22} + ({}_A s_{12} {}_A s_{21} - {}_A s_{11} {}_A s_{22}) = w_A \Gamma_A^{-1}$$

Using (4.4), this becomes

$$\Gamma_A^{-1}{}_A R_{12} - w_A {}_A R_{21} + {}_A R_{11} = w_A \Gamma_A^{-1} {}_A R_{22}$$

or

$$\Gamma_A = \frac{w_A - ({}_A R_{12} / {}_A R_{22})}{({}_A R_{11} / {}_A R_{22})(1 - w_A ({}_A R_{21} / {}_A R_{11}))} \tag{4.15}$$

Comparing this with equation (2.7), we obtain

$$(c_A / d_A) = 1/({}_A R_{11} / {}_A R_{21}) \quad ; \quad d_A = ({}_A R_{11} / {}_A R_{22}) \quad ;$$

$$e_A = ({}_A R_{12} / {}_A R_{22}) \quad ; \tag{4.16}$$

From (4.14) we have now determined (c_A / d_A) and e_A.

Apply (4.3) to transform from Γ_B to w_B instead of from w_A to w_B; this time we must invert Γ_B to allow for the reversed measurement direction and, by the procedure just used, we obtain

$$w_B {}_B s_{11} + \Gamma_B^{-1}{}_B s_{22} + ({}_B s_{12} {}_B s_{21} - {}_B s_{11} {}_B s_{22}) = \Gamma_B^{-1} w_B$$

or

$$w_B {}_B R_{12} - \Gamma_B^{-1}{}_B R_{21} + {}_B R_{11} = \Gamma_B^{-1} w_B {}_B R_{22}$$

or

$$\Gamma_B = \frac{w_B + ({}_B R_{21} / {}_B R_{22})}{({}_B R_{11} / {}_B R_{22})(1 + w_B ({}_B R_{12} / {}_B R_{11}))} \tag{4.17}$$

which gives, by comparison with (2.7) (or with (4.15) and (4.16))

$$(c_B / d_B) = -({}_B R_{12} / {}_B R_{11}) \quad ; \quad d_B = ({}_B R_{11} / {}_B R_{22}) \quad ;$$

$$e_B = -({}_B R_{21} / {}_B R_{22}) \tag{4.18}$$

To obtain information about the quantities in (4.18) we appeal to (4.8); substituting from (4.16) and (4.18) into $^1R = \,^1s_{21}(_AR)(_BR)$ gives

$$
\begin{bmatrix} ^1R_{11} & ^1R_{12} \\ ^1R_{21} & ^1R_{22} \end{bmatrix} = \,^1s_{21} \begin{bmatrix} d_A & e_A \\ c_A & 1 \end{bmatrix}_A R_{22} \begin{bmatrix} d_B & -c_B \\ -e_B & 1 \end{bmatrix}_B R_{22}
$$

If we premultiply this by $(_AR)^{-1}$ in the form

$$
\det(_AR)^{-1}{}_A R_{22} \begin{bmatrix} 1 & -e_A \\ -c_A & d_A \end{bmatrix}
$$

(noting that the physical necessity that $_As_{21}$ and $_As_{12}$ are non-zero means that the determinant is also non-zero), we obtain

$$
\begin{bmatrix} d_B & -c_B \\ -e_B & 1 \end{bmatrix} = (_A R_{22}(^1s_{21}\,{}_B R_{22}\det(_AR))^{-1}) \begin{bmatrix} 1 & -e_A \\ -c_A & d_A \end{bmatrix}\begin{bmatrix} ^1R_{11} & ^1R_{12} \\ ^1R_{21} & ^1R_{22} \end{bmatrix} \quad (4.19)
$$

where all the divisions in the numerical factor are legitimate, including that by $_BR_{22}$ which is $(_Bs_{21})^{-1}$; similarly, since

$$
_A R_{22} = (_As_{21})^{-1} \neq 0
$$

the factor is not zero either. Writing

$$
\theta = \,_A R_{22}(^1s_{21}\,{}_B R_{22}\det(_AR))^{-1}
$$

into (4.19) then gives

$$
-c_B = \theta(^1R_{12} - e_A\,{}^1R_{22}) \quad ; \quad d_B = \theta(^1R_{11} - e_A\,{}^1R_{21}) \quad ;
$$

$$
-e_B = \theta(-c_A\,{}^1R_{11} + d_A\,{}^1R_{21}) \quad ; \quad 1 = \theta(-c_A\,{}^1R_{12} + d_A\,{}^1R_{22})
$$

Dividing out θ,

$$
(c_B/d_B) = -\frac{^1R_{12} - e_A\,{}^1R_{22}}{^1R_{11} - e_A\,{}^1R_{21}} \quad ; \quad d_B = \frac{^1R_{11} - e_A\,{}^1R_{21}}{d_A(^1R_{22} - (c_A/d_A)^1R_{12})} \quad ;
$$

$$
\quad (4.20)
$$

$$
e_B = -\frac{^1R_{21} - (c_A/d_A)^1R_{11}}{^1R_{22} - (c_A/d_A)^1R_{12}}
$$

Since we already know (c_A/d_A) and e_A, equations (4.20), together with the $^1R_{ij}$ defined at (4.8), give us (c_B/d_B), e_B and $(d_B d_A)$ (on multiplying the second of (4.20) by d_A).

From (2.7), (4.15), (4.16), (4.17) and (4.18),

$$\Gamma_A = \frac{w_A - e_A}{d_A(1 - w_A(c_A/d_A))} \quad \text{and} \quad \Gamma_B = \frac{w_B - e_B}{d_B(1 - w_B(c_B/d_B))} \quad (4.21)$$

with all the parameters known except d_A and d_B whose product is known.

Multiplying equations (4.21) together gives an expression for $(\Gamma_A \Gamma_B)$ which is completely determined (since the factors d_A and d_B combine into their known product and everything else is already known). We now invoke condition 3 of the calibration procedure in which the same fixed load is presented first to one measurement port and then to the other. The Γ_A of one of these observations is clearly equal to the Γ_B of the other (and vice versa, since the realisation in calibration step d consists of two fixed loads which are exchanged) and their product is Γ_x^2, where Γ_x is the VRC of the relevant termination. A trivial amount of information about the nature of the fixed termination(s) is sufficient to determine the correct square root of Γ_x^2 (eg if $\Gamma_x^2 \cong 1 + j0$ one can distinguish between short- and open-circuits by inspection). Then, using equations (4.21) separately d_A and d_B can be found (and measurements on both fixed terminations provide for a check by comparison of results). The c_A, d_A, e_A and c_B, d_B, e_B characterise the two reflectometers so that calibration is now complete.

The Γ_A or Γ_B at the measurement port of each six-port reflectometer can be found from the observed Q_k by determining α, β and γ from equation (3.6), finding the appropriate w from (3.13) (since the appropriate sign for v is known), and finally obtaining Γ from (4.21). The Γ_{Ai} and Γ_{Bi} obtained for the settings of the adjustable phase—changer can then be used to set up a set of at least three simultaneous inhomogeneous equations (4.2) which are solved by standard methods for s_{11}, s_{22} and the product $s_{12}s_{21}$ of a two-port junction. If this is reciprocal in the sense that $s_{12} = s_{21}$ then it can be completely characterised provided plausible arguments, like those of section 4.2, may be relied on to resolve the square-root ambiguity (but see sections 4.4 and 4.5).

The calibration procedure of section 4.1 and Figure 4 provides for these measurements on all the one- and two-port junctions used and the results serve to monitor the consistency of calibration.

4.4 CALIBRATION THEORY FOR MEASURING $|s_{12}|$ AND $|s_{21}|$ OF A NON-RECIPROCAL TWO-PORT JUNCTION TRANSMITTING THE SAME MODE AT BOTH PORTS

So far, we have not made use of the seventh power ratio $Q_{AB} = P_{BR}/P_{AR}$ defined at (4.5); this provides another calibration constant which enables $|s_{12}|$ and $|s_{21}|$ of a two-port junction to be determined. (Thus the assumption of reciprocity $s_{12}s_{21} = 1$ can be verified within an ambiguity of an even multiple of π, which should be resolvable from physical considerations.)

From equation (3.20) the power flux from the measurement port of reflectometer A is

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$$P_A = G_o(1-|\Gamma_A|^2)|b_A|^2 \qquad (4.22)$$

But from (3.22) and the definitions at (4.5)

$$P_A = K_A P_{AR}(|d_A-w_A c_A|^2 - |w_A-e_A|^2)$$

Substituting for Γ_A from (4.21) this becomes

$$P_A = K_A P_{AR}|d_A-w_A c_A|^2(1-|\Gamma_A|^2)$$

$$\therefore \qquad |b_A|^2 = K_A P_{AR}|d_A-w_A c_A|^2/G_o \qquad \text{(from (4.22))}$$

Evaluating P_B, the power flux from measurement port B, similarly gives

$$\left|\frac{b_A}{b_B}\right|^2 = \frac{K_A}{K_B}\left|\frac{d_A-w_A c_A}{d_B-w_B c_B}\right|^2 \frac{P_{AR}}{P_{BR}} \qquad (4.23)$$

But (K_A/K_B) is a characteristic of the network analyser and can be found from the observations of the power ratio Q_{AB} when the two reflectometers are connected together (calibration condition 1) for then, using primes in equation (4.22) to indicate this condition

$$\left|\frac{b_A'}{b_B'}\right|^2 = \frac{1-|\Gamma_B'|^2}{1-|\Gamma_A'|^2} \frac{P_A'}{P_B'}$$

But $P_A' = -P_B'$ so that, from this equation, (4.21) and (4.23)

$$\frac{K_A}{K_B} = -\frac{P_{BR}'}{P_{AR}'}\left(\frac{|d_B-w_B'c_B|^2 - |w_B'-e_B|^2}{|d_A-w_A'c_A|^2 - |w_A'-e_A|^2}\right) \qquad (4.24)$$

Thus (K_A/K_B) can be found by observing $Q_{AB}' = P_{BR}'/P_{AR}'$ (as in (4.5)) during calibration condition 1. Since it is a constant for the network analyser, and since observations are made with at least three settings of the phase-changer, it provides another check of calibration consistency. (The usual least-squares solution for the best value conveniently reduces to the arithmetic mean of the separately calculated values.)

From equations (4.1) and (4.2)

$$\Gamma_A = \frac{a_A}{b_A} = s_{11} + s_{12}\frac{b_B}{b_A} \quad \text{and} \quad \Gamma_B = \frac{a_B}{b_B} = s_{22} + s_{21}\frac{b_A}{b_B}$$

$$\therefore \qquad s_{12} = (\Gamma_A-s_{11})\frac{b_A}{b_B} \quad \text{and} \quad s_{21} = (\Gamma_B-s_{22})\frac{b_B}{b_A} \qquad (4.25)$$

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$$\therefore \qquad |s_{12}| = |\Gamma_A - s_{11}| \left|\frac{b_A}{b_B}\right| \quad \text{and} \quad |s_{21}| = |\Gamma_B - s_{22}| \left|\frac{b_B}{b_A}\right| \qquad (4.26)$$

Having found s_{11} and s_{22} from equation (4.3), we have at least three sets of Γ_A, Γ_B and of $|b_A/b_B|$ using (4.23), since no ambiguity of sign arises from this (because the sign of a modulus is positive). Hence $|s_{12}|$ and $|s_{21}|$ can each be calculated several times from equation (4.26) and their average values taken (as for calculating (K_A/K_B)). We can thus measure $|s_{12}|$ and $|s_{21}|$ of a two-port junction transmitting the same mode at both ports (whether $s_{12} = s_{21}$ or $s_{12} \neq s_{21}$).

4.5 DIRECT MEASUREMENT OF ARGUMENTS $\angle s_{12}$ and $\angle s_{21}$

In principle, the phase angles $\angle s_{12}$ and $\angle s_{21}$ can be measured but this process has not been implemented at RSRE because this measurement has not been required and because the network analyser of Figure 3 has to be modified to include isolators (which are temperature-dependent components and which would limit the bandwidth of some reflectometers). Nevertheless, we include a derivation of the method, following (19), both for the sake of completeness and to clarify the additional assumptions necessary for this measurement to be possible.

The modified network analyser would take the form illustrated in Figure 6, for which it must be assumed that the phase-shifter is precisely
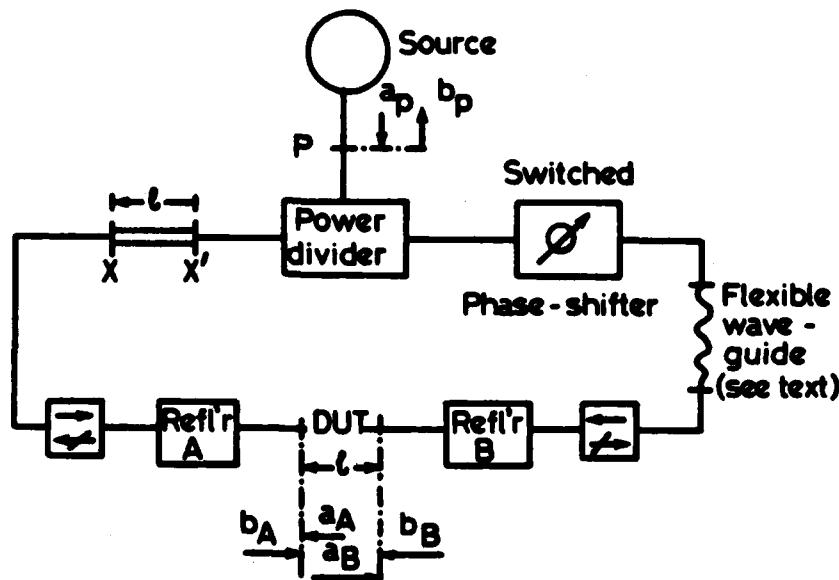

FIG.6. DUAL NETWORK ANALYSER SUITABLE FOR MEASURING $\angle s_{12}$ AND $\angle s_{21}$ OF DUT

repeatable for each series of power ratio observations. It is also assumed that, after the reflectometers have been calibrated by the methods already described and after measurements have been made of the scattering coefficients of three reciprocal two-port junctions, then the flexible waveguide is replaced by a rigid one. These three reciprocal junctions must each be of the same length ℓ as the non-reciprocal two-port DUT; the $\angle s_{12}(= \angle s_{21})$ of two of them must differ and the third must be a non-reflecting line. This line must be capable of being connected at XX' in Figure 6 (ie between the power divider and one isolator) so that the connectors forming this reference plane must be of the same form as those of the reflectometer measurement ports.

We approach the measurement problem by considering the entire network analyser as a three-port junction, of which two ports are the reflectometer measurement ports and the third is a reference plane P situated between the power divider and the source, as shown in Figure 6. We can write the scattering equations for this junction as

$$\begin{bmatrix} b_A \\ b_B \\ b_P \end{bmatrix} = \begin{bmatrix} s_{AA} & s_{AB} & s_{AP} \\ s_{BA} & s_{BB} & s_{BP} \\ s_{PA} & s_{PB} & s_{PP} \end{bmatrix} \begin{bmatrix} a_A \\ a_B \\ a_P \end{bmatrix}$$

and the first two of these equations are

$$b_A = s_{AA}a_A + s_{AB}a_B + s_{AP}a_P \quad \text{and} \quad b_B = s_{BA}a_A + s_{BB}a_B + s_{BP}a_P$$

so that a_P can be eliminated between them giving

$$b_A = s_{AA}a_A + s_{AB}a_B + s_{AP}(1/s_{BP})(b_B - s_{BA}a_A - s_{BB}a_B)$$

But the VRC measured by the reflectometers are $\Gamma_A = a_A/b_A$ and $\Gamma_B = a_B/b_B$ so that

$$\frac{b_A}{b_B} = \left\{ \frac{s_{AP}}{s_{BP}} \right\} + \left\{ s_{AA} - s_{BA} \frac{s_{AP}}{s_{BP}} \right\} \frac{b_A}{b_B} \Gamma_A + \left\{ s_{AB} - s_{BB} \frac{s_{AP}}{s_{BP}} \right\} \Gamma_B \qquad (4.27)$$

Now the quantities in braces in equation (4.27) are characteristics of the network analyser and if they can be found by calibration then b_A/b_B can be calculated from Γ_A and Γ_B for each phase-shifter setting allowing s_{12} and s_{21} to be completely determined from equation (4.25).

Calibration of the network analyser for measuring $\angle s_{12}$ and $\angle s_{21}$ thus involves calibrating the repeatable phase shifter and the rest of the analyser and it is necessary for this that any change of phase between b_A and b_B caused by physically separating the two reflectometers be known. Assuming sufficient isolation by the two isolators of Figure 6 for the approximation $s_{AB} = s_{BA} = 0$ to be valid, then s_{AA} and s_{BB} will not change when the reflectionless line is inserted at XX' of Figure 6 to allow separation of the reflectometers and s_{AP} changes by the factor $e^{-\delta \ell}$, where δ is the known propagation constant of the line (as does s_{PA}). (This can be shown by analysing the network of power divider, isolators etc forming the three-port junction.)

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Given this assumption of sufficient isolation and the approximations resulting from it, the two reflectometer measurement ports can be connected to each other, so that $b_A/b_B = \Gamma_B$, and we have from (4.27) that

$$\Gamma_B = A_1 + A_2 - A_3\Gamma_B$$

where $\quad A_1 = \{s_{AP}/s_{BP}\}$; $\quad A_2 = \{s_{AA}\}$; $\quad A_3 = \{s_{BB}s_{AP}/s_{BP}\}$.

By connecting in turn the two known two-port reciprocal junctions between the reflectometer ports (for which we also need to connect the known line at XX'), we can provide from equation (4.25) two more known $(b_A/b_B)_m$, (m = 1,2) so that from (4.27) we have

$$(b_A/b_B)_m = A_1 e^{-\delta\ell} + A_2(b_A/b_B)_m \Gamma_{Am} - A_3 e^{-\delta\ell}\Gamma_{Bm}$$

Hence, we can obtain a set of simultaneous equations in the unknowns A_1, A_2, A_3 for each phase-shifter setting. Assuming this repeats, this further stage of calibration allows equation (4.25) to be used to obtain s_{12} and s_{21} completely for a non-reciprocal two-port junction.

We remark that values of s_{12} and s_{21} so obtained are most suitable for determining $\angle s_{12}$ and $\angle s_{21}$. The magnitudes, $|s_{12}|$ and $|s_{21}|$, obtained from (4.26) are likely to be more accurate because they do not rely on the phase-shifter being repeatable or on the isolation being sufficient. Consistency between the magnitudes obtained by (4.25) and (4.26) may provide a verification of these assumptions. We also remark that this method provides a method of verifying the plausibility of arguments used in section 4.3 for resolving the ambiguity of sign of the square root of $(s_{12})^2$ for a reciprocal two-port; even with the perhaps dubious assumptions made concerning the repeatability and predictability of the three-port junction comprising the analyser of Figure 6, the method should be able to resolve an ambiguity of 0 or π.

4.6 SIMULTANEOUS COMPARISON OF POWER STANDARDS

We have shown in section 3.4 that power standards can be compared by connecting them in turn to the measurement port of a six-port reflectometer and using equation (3.22). We note that a dual reflectometer network analyser allows equation (4.24) to be used for comparing two power standards simultaneously (rather than sequentially) and their VRCs can be also determined.

5 AIR DIELECTRIC TRANSMISSION LINES AS IMPEDANCE STANDARDS

The assumptions made in our theory for calibrating the dual reflectometer are that:

a.   the source provides single-frequency radiation;

b.   each detector provides an indication proportional to the power it absorbs;

c.   the reflectometer junction transmits only one mode at each port and that both measurement ports transmit the same mode;

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d.   the uniform line forming the calibration standard has measurable
attenuation and connectors at each end of a type that permit its length
to be accurately measured.

Then, to the extent that the connectors at the ends of the standard line may be
considered electrically "invisible", both reflectometers (including the connec-
tors forming their measurement ports) are calibrated in terms of the impedance
Z_0 characterising the mode transmitted by the standard line. Connectors to
realise condition (d) for rectangular waveguide are relatively easily implemented
by lapped, dowelled, waveguide flanges; and, for some sizes of air-dielectric
precision coaxial line, connectors are available providing a good approximation
to coplanar inner- and outer-conductor contacts. For both, the contact impedance
is included in the attenuation constant δ measured of the standard during cali-
bration, but this inclusion is unfortunately not self-consistent (because when
the measurement ports are connected together only one connector pair is involved,
whereas connection through the standard line involves two pairs).

At some frequencies, the imperfect surface finish of coaxial line conductors
causes Z_0 to depart measurably from the nominal characteristic impedance R_0 (real)
calculable for the TEM mode for an ideal lossless line of the same dimensions.
In this event, the measured VRC Γ_A and Γ_B may be renormalised to the nominal
characteristic impedance R_0. The transmission-line model of a two-conductor
line describes it in terms of series resistance R and inductance L, shunt
conductance G and capacitance C, all per unit length at some particular angular
frequency ω; then

$$Z_0 = \sqrt{(R+j\omega L)/(G+j\omega C)} \quad \text{and} \quad \delta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

where δ may be identified as the true propagation constant of the line as defined
in section 3.3 and Z_0 is the impedance characterising the line. The branches of
the square roots chosen are, of course, those which make Z_0 positive-real and δ
positive-imaginary for small R and G so that $x = \exp(-2\delta\ell)$ in section 3.3. Hence
on rearranging we have $Z_0 = \delta/(G+j\omega C)$ where C can be identified as the calculable
electrostatic capacitance per unit length of the precision line. For air
dielectric we may accurately set $G = 0$ so that Z_0 may be determined from the
measured δ and the calculable C. Moreover

$$R_0 = \frac{1}{v_0 C}$$

where v_0 is the velocity of propagation so that the indicated Γ (normalised to
Z_0) may be renormalised to R_0 (to give Γ_0) since

$$Z_0 \frac{1+\Gamma}{1-\Gamma} = R_0 \frac{1+\Gamma_0}{1-\Gamma_0}$$

ie

$$\Gamma_0 = \frac{\Gamma(1+Z_0/R_0)-(1-Z_0/R_0)}{(1+Z_0/R_0)-\Gamma(1-Z_0/R_0)}$$

Although this process of renormalisation to a standard R_0 calculable from
mechanical dimensions and the velocity of propagation in air is attractive as a
means of standardisation, it will require subsidiary measurement of the "residual"
contact impedance of the air-line connectors as this enters into the propagation

constant measured. The resolution of the six-port reflectometers should enable these to be applied in established methods of measurement of this contact impedance (16).

6   SIMULTANEOUS-EQUATION-SOLVING ALGORITHMS

6.1 LEAST-SQUARES SOLUTION FOR REAL UNKNOWNS

This section outlines the method employed to refine the provisional solutions of simultaneous non-linear equations appearing in the six- to four-port reduction derived in section 3.1. The methods are outlined here because most desktop computers have no library of suitable routines. One stage of the method entails solving an overdetermined set of linear equations. The solution of these is discussed separately here because the same problem occurs in calibrating either a four- or a six-port reflectometer by the use of known loads, derived in sections 2.2 and 3.2, respectively.

Suppose we have a set of M equations, not necessarily linear, in N ($\le$ M) unknowns; thus

$$f_i(x_1, x_2, \ldots, x_j, \ldots x_N) = 0 \quad \text{for } i = 1, \ldots, M \quad (6.1)$$

If we have approximations ^0x_j to the unknowns, we may re-write (6.1), neglecting second-order terms, as

$$f_i(^0x_1, {}^0x_2 \ldots, {}^0x_N) + \sum_{j=1}^{N} (^1x_j - {}^0x_j) \frac{\partial f_i}{\partial x_j} = 0 \text{ for } i = 1, \ldots, M \quad (6.2)$$

where all the partial derivatives are evaluated at the point ($x_1 = {}^0x_1, \ldots, x_j = {}^0x_j, \ldots, x_N = {}^0x_N$). This set of equations is linear in the corrections ($^1x_j - {}^0x_j$), and may be solved for them by the method described below. The correction process may be repeated as often as necessary; it displays quadratic convergence, like Newton's method of which it is a generalisation.

Suppose now that we have a set of M linear equations in N ($\le$ M) unknowns; in matrix notation we may write

$$Ax = b \quad (6.3)$$

where A contains the coefficients and is of order (M x N), x contains the unknowns and is of order (N x 1), and b contains the right-hand sides and is of order (M x 1). We assume for the moment that A, x and b are real; this restriction will be removed later.

In the special case where M = N, (6.3) admits the solution

$$x = A^{-1}b \quad (6.4)$$

When M > N, it is customary to choose the solution x such that the scalar quantity $(Ax-b)^T(Ax-b)$ is an absolute minimum, where T denotes matrix transpose. This is the "method of least-squares", first adopted by Gauss in the light of his error distribution but now routinely used because of its formal simplicity.

To minimise $(Ax-b)^T(Ax-b)$ absolutely, we differentiate partially with respect to each component of x and set all the resulting quantities equal to zero. We then obtain N simultaneous linear equations in the N components of x, and they admit the formal solution

$$x = (A^TA)^{-1}A^Tb \qquad (6.5)$$

When $M = N$, (6.5) reduces to (6.4), the absolute minimum of $(Ax-b)^T(Ax-b)$ then being zero.

When $M > N$, (6.5) is not numerically-stable; the three appearances of A (compared to the one appearance in (6.4)) cause a large increase in the rounding errors during the actual solution. This problem is overcome by resolving the matrix A into a product of matrices Q and U, where the matrix Q (of order (M x N)) has the property that $Q^TQ = I_N$ (the identity matrix of order (N x N)), and the matrix U is of order (N x N) and can be inverted easily. From (6.5) we then have

$$x = U^{-1}Q^Tb \qquad (6.6)$$

which is a numerically-stable form for x because the components of A only appear once instead of three times as in (6.5). The resolution is brought about by applying to A N successive transformations of the kind named after Householder, which are described by Fox (17). The matrix U is upper-triangular (it has non-zero elements only on and above the diagonal from its top left corner to its bottom right corner) which makes it easy to find x from (6.6) or from the equivalent form

$$Ux = Q^Tb \quad . \qquad (6.7)$$

During the resolution it is usually necessary to alter the order of the rows of A; this process, and the later unscrambling of it, will also be found in Fox's book.

6.2 EXTENSION TO COMPLEX UNKNOWNS

We consider now the extension of this method to complex A, x and b. If j is $\sqrt{-1}$, we may write in an obvious notation

$$A = A_r + jA_i \quad ; \quad x = x_r + jx_i \quad ; \quad b = b_r + jb_i \qquad (6.8)$$

Then, from (6.3) and (6.8), we have

$$(A_rx_r - A_ix_i) + j(A_rx_i + A_ix_r) = (b_r) + j(b_i) \qquad (6.9)$$

If we equate real and imaginary parts in (6.9), we may re-write it in the partitioned-matrix form

$$\begin{pmatrix} A_r & -A_i \\ \hline A_i & A_r \end{pmatrix} \begin{pmatrix} x_r \\ \hline x_i \end{pmatrix} = \begin{pmatrix} b_r \\ \hline b_i \end{pmatrix} \qquad (6.10)$$

which is purely real and may be solved using the method of the previous section. In the real case, minimising $(Ax-b)^T(Ax-b)$ is the same as minimising the sum of the squares of the differences between the left- and right-hand sides of each equation; when a complex problem is attacked using (6.10), we minimise the sum of the squares of the real and imaginary parts of the differences, which is the sum of the squares of their moduli. For uncorrelated equations this can be shown to be correct from probability considerations. It is, apparently, unsatisfactory because the phases of the differences are not constrained by minimising the sum of squared moduli; but by considering Figure 7 we can see that this does not matter. We have a right-hand side vector b added to a difference vector δ; if $|\delta| \ll |b|$, then, wherever the direction of δ may be, the sum vector s is still tightly-constrained both in magnitude and in direction. Moreover, the maximum difference in phase between s and b corresponds to the minimum difference in magnitude, and vice versa, so the differences are well-balanced in practice.

The sum of b and δ lies on the dotted circle, and is always tightly-constrained. A similar conclusion is valid for multi-dimensional vectors

FIG. 7.

7 CONCLUSION

It is axiomatic that the quality of a product or service cannot be maintained, let alone improved, without means of measuring that quality. This is true also for measurement processes which require repeated (and therefore in practice automated) measurements for their variation, and the stability of the reference standards, to be quantified. In principle, the calibration methods treated in this report allow the stability of six-port reflectometer instrumentation to be checked without reference to standards whose long-term stability would have to be inferred or even assumed. Moreover, the resolution obtained at RSRE of measurement of VRC with these methods suggests that the long-term stability of the reference standards can be compared since each calibration requires only one such standard.

The derivation of the theory of these six-port reflectometer calibration and measurement methods has been presented in this report as an analytic deduction from the basic postulates of linearity of the reflectometer and its detectors. This was a necessary prerequisite for programming the solutions to the problems (which are trivial only in the mathematical sense that they have been solved), especially to ensure that all the ambiguities of square-root sign are resolved. This approach appears to the authors to be more appropriate for computation than that contained in earlier treatments (relying on geometric interpretations of diagrams in the complex plane of six-port reflectometer behaviour).

8 ACKNOWLEDGEMENTS

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four-port reflectometers, we can now derive the theory for their calibration.

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